National Technical University of Athens
Department of Mechanics

Nonlinear Dynamics of Hysteretic Mechanical Systems: Some Recent Advancements

Part 1
The Vaiana-Rosati Model of Hysteresis

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Introduction

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Research Group of Prof. Luciano Rosati

Introduction

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Main Research Topics

Mechanics of materials and structures

engineering materials (elastomers and metals)

engineering devices and structures

Computational mechanics

numerical methods

(iterative procedures, integration methods)

Dynamics and control

dynamics of solids and structures

(hysteretic and viscous models)

nonlinear dynamics

(dynamic systems, numerical methods in dynamics)

Control

isolation of structures; dissipative control systems

Agenda

Part A

Classification

Complex hysteresis loops

Classification of complex hysteresis loops

Modeling of complex hysteresis loops
- Review of a generalized class of models
- Vaiana-Rosati model

Validation of the Vaiana-Rosati model
- Validation against experimental results
- Validation against numerical results

Reformulation of the Vaiana-Rosati model
- Analytical reformulation (VRM+A)
- Differential reformulation (VRM+D)
- VRM+A versus VRM+D

Agenda

Part B

Phenomenological Modeling

Complex hysteresis loops

Classification of complex hysteresis loops

Modeling of complex hysteresis loops
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Validation of the Vaiana-Rosati model
- Validation against experimental results
- Validation against numerical results

Reformulation of the Vaiana-Rosati model
- Analytical reformulation (VRM+A)
- Differential reformulation (VRM+D)
- VRM+A versus VRM+D

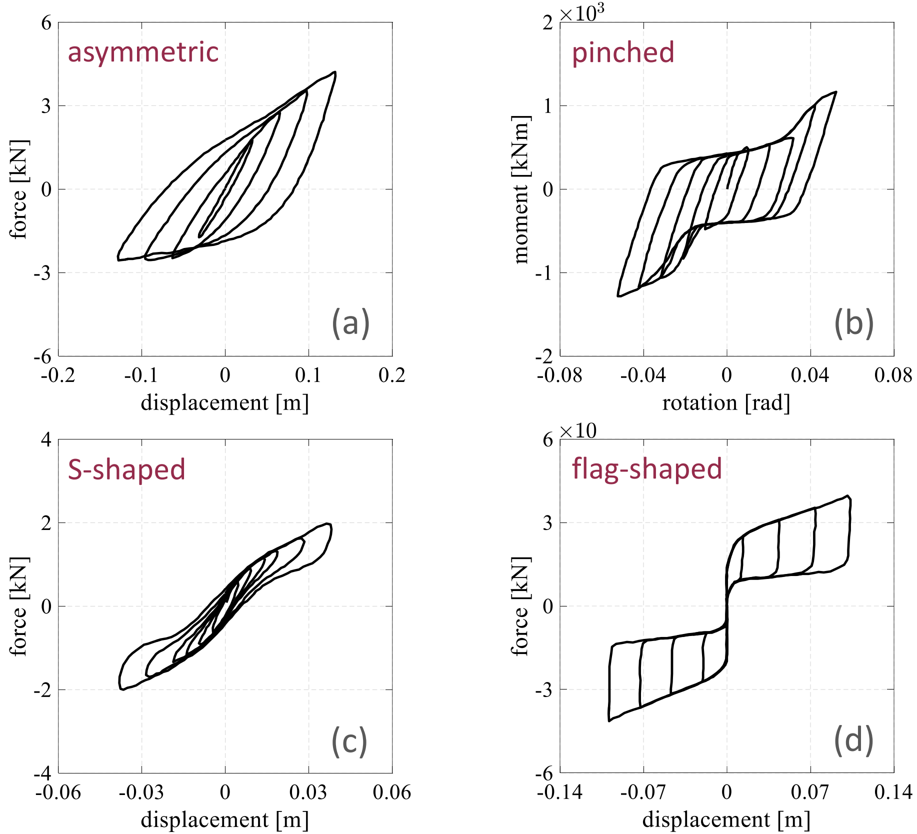
Complex hysteresis loops

(a) **flexible connector** for electrical substations (Filiatrault and Kremmidas 2000)

(b) **steel beam-column connection** (Kim et al. 2012)

(c) **unbonded fiber-reinforced elastomeric bearing** (Manzoori and Toopchi-Nezhad 2017)

(d) **rocking timber wall with friction dampers** (Hashemi et al. 2020)



Classification of complex hysteresis loops

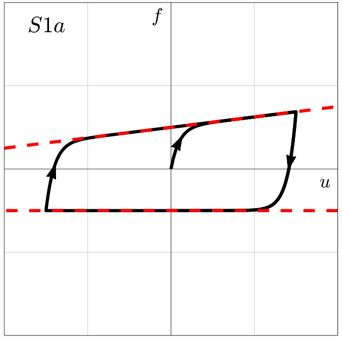
Hysteresis loops limited by:

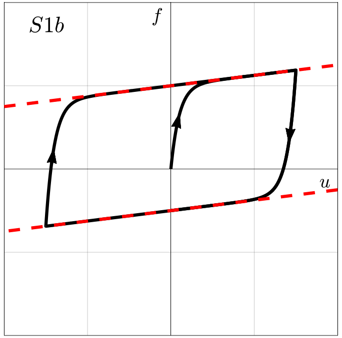
S1) two straight lines

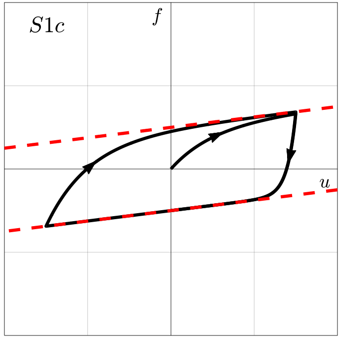
S2) two curves with no inflection point

S3) two curves with one inflection point

S4) two curves with two inflection points







smooth steel reinforcing bars

steel dampers (shear link device)

steel beam-column connections

Classification of complex hysteresis loops

Hysteresis loops limited by:

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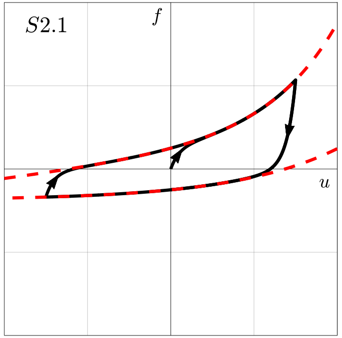
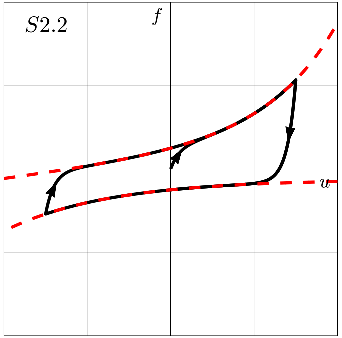
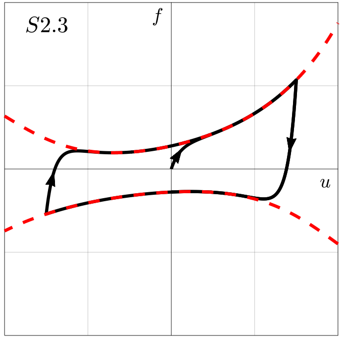
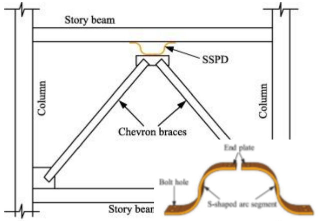






wire rope isolators

expansion anchors



steel dampers

Classification of complex hysteresis loops

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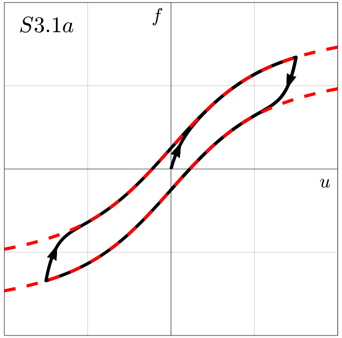

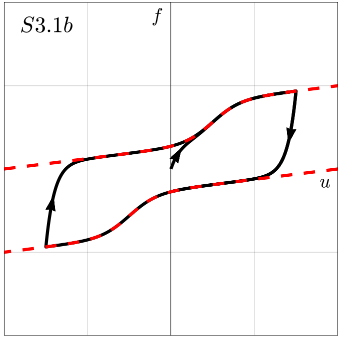

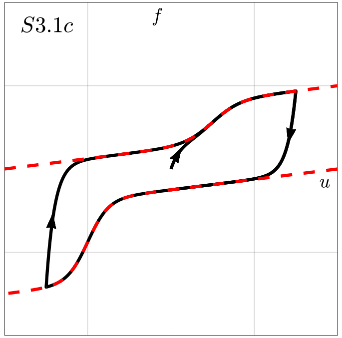

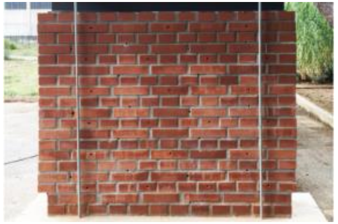

brick masonry walls

toe-screwed wood connections

wood shear walls

Classification of complex hysteresis loops

Hysteresis loops limited by:

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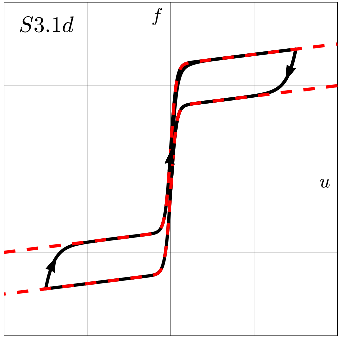
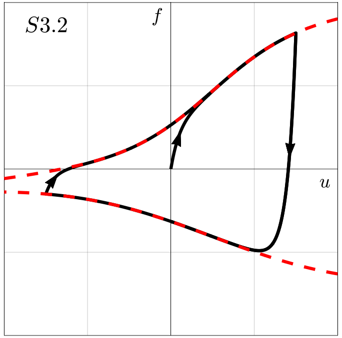
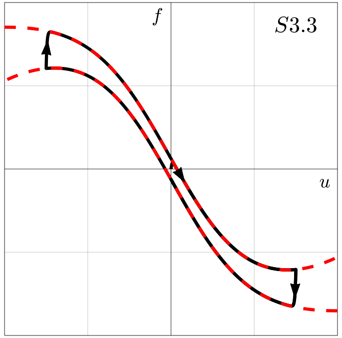
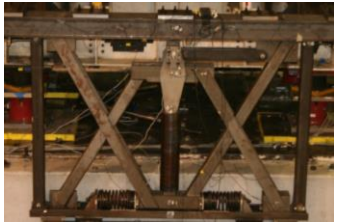






SMA helical springs

buckling steel braces



negative stiffness devices

Classification of complex hysteresis loops

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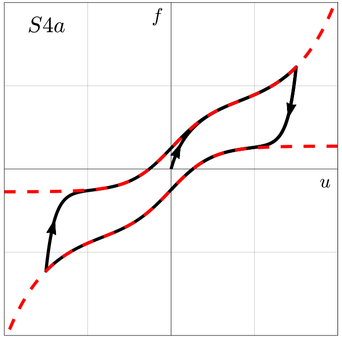
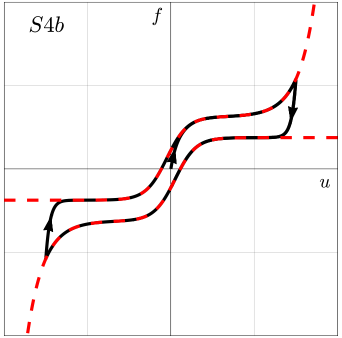
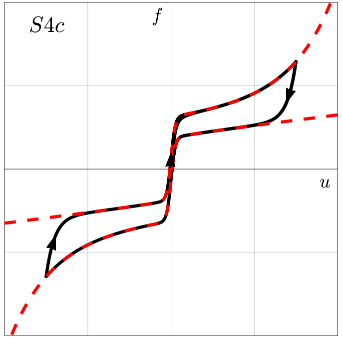
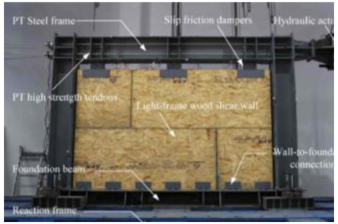






fiber reinforced rubber bearings

reinforced concrete walls



steel-timber hybrid shear walls

Modeling of complex hysteresis loops

Review of a generalized class of models

ELSEVIER
Mechanical Systems and Signal Processing
Volume 146, 1 January 2021, 106984

A generalized class of uniaxial rate-independent models for simulating asymmetric mechanical hysteresis phenomena

Nicolò Vaiana, Salvatore Sessa, Luciano Rosati

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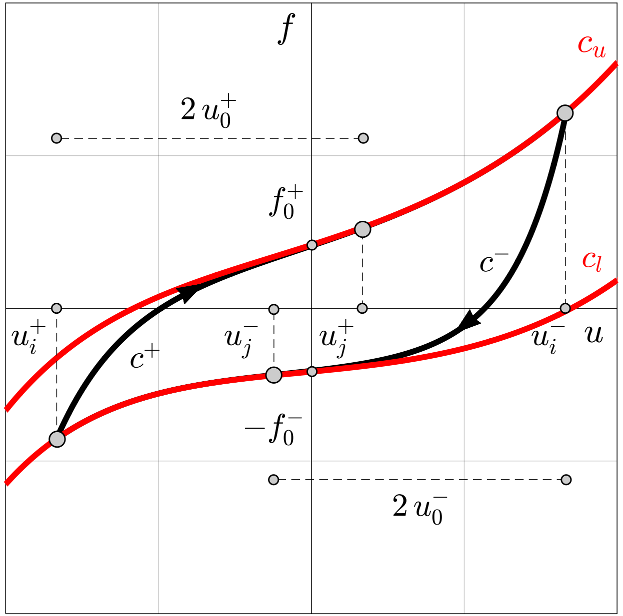
https://doi.org/10.1016/j.ymssp.2020.106984

Modeling of complex hysteresis loops

Review of a generalized class of models

f output variable

u input variable

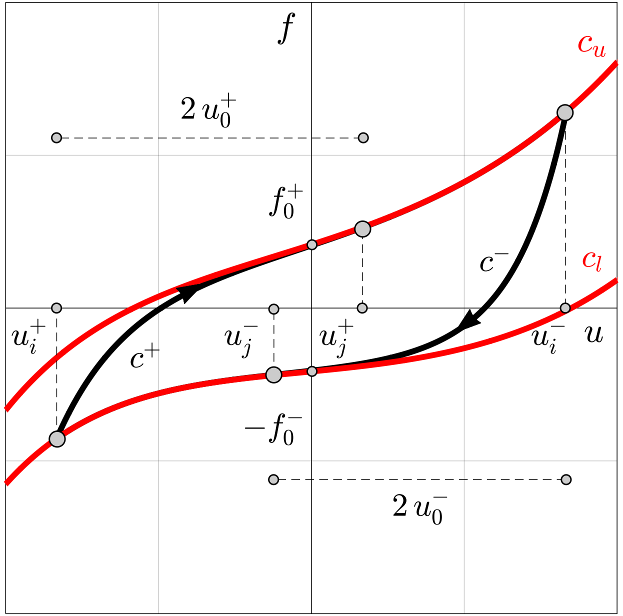


Modeling of complex hysteresis loops

Review of a generalized class of models

$$f(u, {u_j}^+) = \begin{cases} c^+(u, {u_j}^+) & u < {u_j}^+ \\ c_u\,(u) & u > {u_j}^+ \end{cases}$$

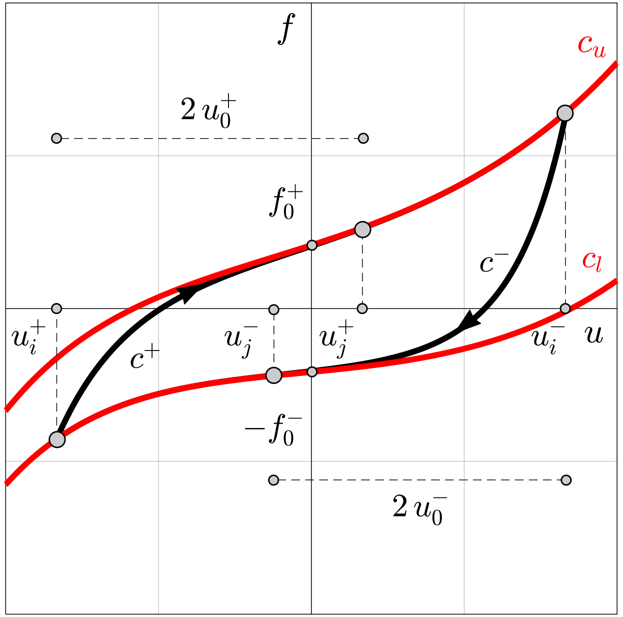
$$f(u, {u_j}^-) = \begin{cases} c^-(u, {u_j}^-) & u > {u_j}^- \\ c_l\,(u) & u < {u_j}^- \end{cases}$$



Modeling of complex hysteresis loops

Review of a generalized class of models

c^+	generic loading curve
c_u	upper limiting curve
c^-	generic unloading curve
c_l	lower limiting curve

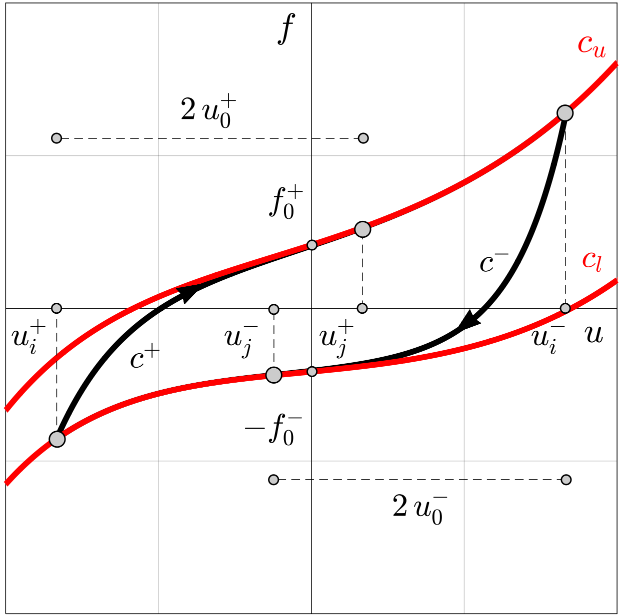


Modeling of complex hysteresis loops

Review of a generalized class of models

$u_j{}^+$ internal variable (loading phase)

$u_j{}^-$ internal variable (unloading phase)



Modeling of complex hysteresis loops

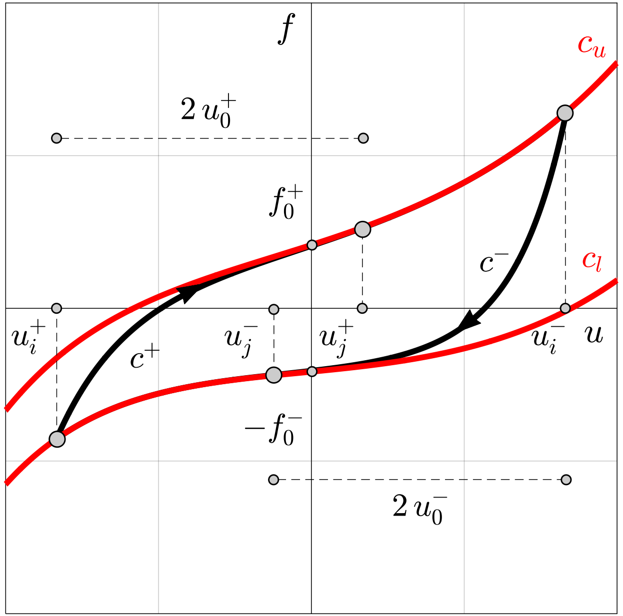
Review of a generalized class of models

$f_0{}^+$ model parameter

$u_0{}^+$ model parameter

$f_0{}^-$ model parameter

$u_0{}^-$ model parameter



Modeling of complex hysteresis loops

Vaiana-Rosati model

ELSEVIER

Mechanical Systems and Signal Processing

Volume 182, 1 January 2023, 109539

Classification and unified phenomenological modeling of complex uniaxial rate-independent hysteretic responses

Nicolò Vaiana, Luciano Rosati

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https://doi.org/10.1016/j.ymssp.2022.109539

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Modeling of complex hysteresis loops

Vaiana-Rosati model

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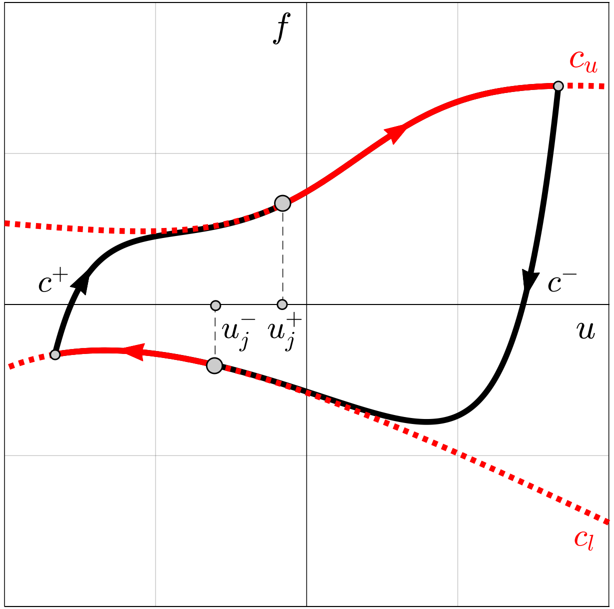
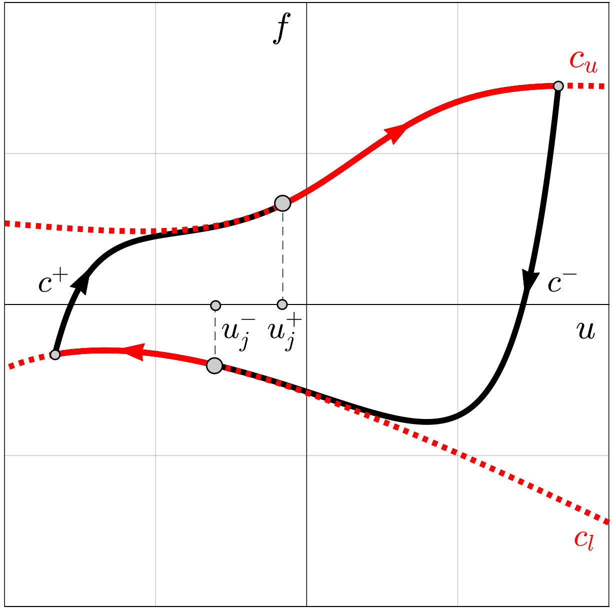


Modeling of complex hysteresis loops

Vaiana-Rosati model

$$c^{+}(u, {u_j}^{+}) = {f_e}^{+}(u) + {k_b}^{+}u + {f_0}^{+} - \frac{1}{\alpha^{+}}\left[e^{-\alpha^{+}(+u-{u_j}^{+}+\bar{u}^{+})} - e^{-\alpha^{+}\bar{u}^{+}}\right]$$

$$c^{-}(u, {u_j}^{-}) = {f_e}^{-}(u) + {k_b}^{-}u - {f_0}^{-} + \frac{1}{\alpha^{-}}\left[e^{-\alpha^{-}(-u+{u_j}^{-}+\bar{u}^{-})} - e^{-\alpha^{-}\bar{u}^{-}}\right]$$

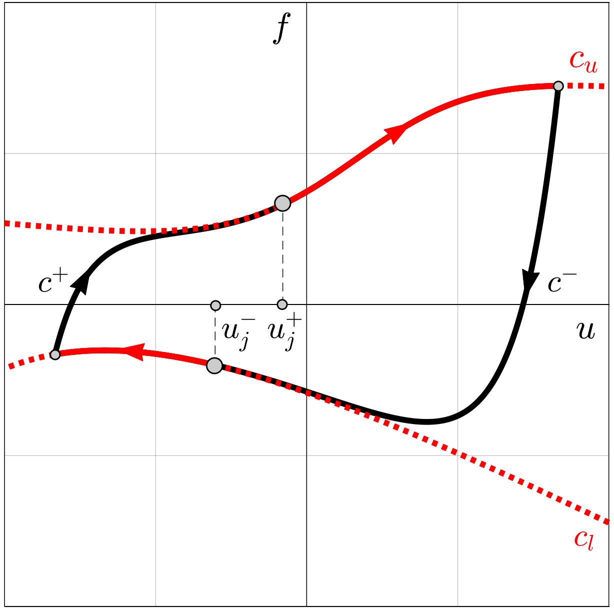


Modeling of complex hysteresis loops

Vaiana-Rosati model

$$c_u\,(u) = f_e{}^{+}(u) + k_b{}^{+}u + f_0{}^{+}$$

$$c_l\,(u) = f_e{}^{-}(u) + k_b{}^{-}u - f_0{}^{-}$$

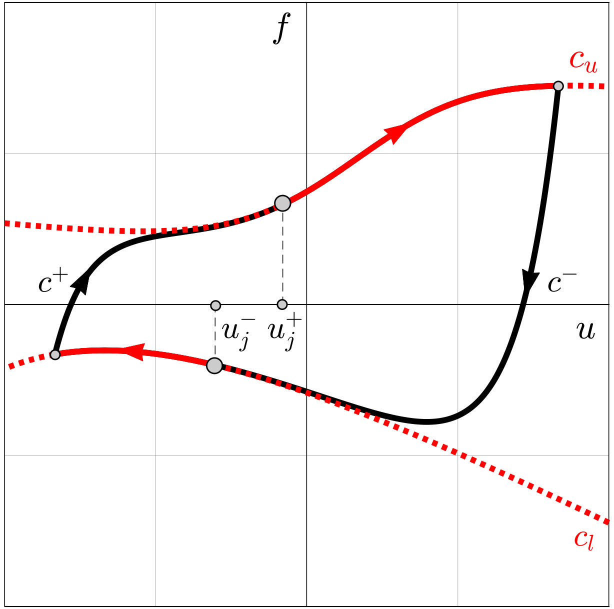


Modeling of complex hysteresis loops

Vaiana-Rosati model

$$f_e^{+}(u) = \beta_1^{+} e^{\beta_2^{+} u} - \beta_1^{+} + \frac{4\,\gamma_1^{+}}{1 + e^{-\gamma_2^{+}(u-\gamma_3^{+})}} - 2\gamma_1^{+}$$

$$f_e^{-}(u) = \beta_1^{-} e^{\beta_2^{-} u} - \beta_1^{-} + \frac{4\,\gamma_1^{-}}{1 + e^{-\gamma_2^{-}(u-\gamma_3^{-})}} - 2\gamma_1^{-}$$

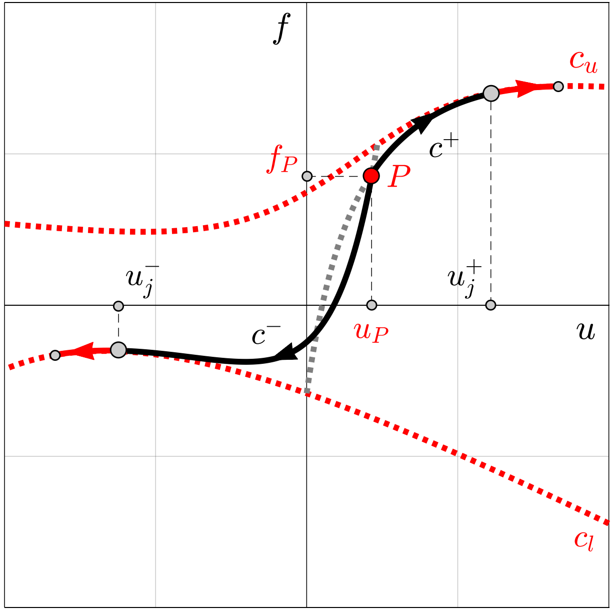


Modeling of complex hysteresis loops

Vaiana-Rosati model

$$u_j{}^{+} = u_P + \bar{u}^{+} + \frac{1}{\alpha^{+}} \ln\left\{+\alpha^{+}\left[f_e{}^{+}(u_P) + k_b{}^{+} u_P + f_0{}^{+} + \frac{1}{\alpha^{+}} e^{-\alpha^{+}\bar{u}^{+}} - f_P\right]\right\}$$

$$u_j{}^{-} = u_P - \bar{u}^{-} - \frac{1}{\alpha^{-}} \ln\left\{-\alpha^{-}\left[f_e{}^{-}(u_P) + k_b{}^{-} u_P - f_0{}^{-} - \frac{1}{\alpha^{-}} e^{-\alpha^{-}\bar{u}^{-}} - f_P\right]\right\}$$



Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

$k_b^+ \quad f_0^+ \quad \alpha^+$

$\beta_1^+ \quad \beta_2^+ \quad \gamma_1^+ \quad \gamma_2^+ \quad \gamma_3^+$

unloading phase

$k_b^- \quad f_0^- \quad \alpha^-$

$\beta_1^- \quad \beta_2^- \quad \gamma^- \quad \gamma_2^- \quad \gamma_3^-$

shape type	limiting curves	subtype	obtained for	
$S\,1$	straight lines	–	$\beta_1^+ = \beta_2^+ = 0$ $\beta_1^- = \beta_2^- = 0$	$\gamma_1^+ = \gamma_2^+ = 0$ $\gamma_1^- = \gamma_2^- = 0$
$S\,2$	curves with no inflection point	$S\,2.1$	$\beta_1^+ > 0,\ \beta_2^+ > 0$ $\beta_1^- > 0,\ \beta_2^- > 0$	$\gamma_1^+ = \gamma_2^+ = 0$ $\gamma_1^- = \gamma_2^- = 0$
		$S\,2.2$	$\beta_1^+ > 0,\ \beta_2^+ > 0$ $\beta_1^- < 0,\ \beta_2^- < 0$	$\gamma_1^+ = \gamma_2^+ = 0$ $\gamma_1^- = \gamma_2^- = 0$
		$S\,2.3$	$\beta_1^+ > 0,\ \beta_2^+ > 0$ $\beta_1^- < 0,\ \beta_2^- < 0$	$\gamma_1^+ > 0,\ \gamma_2^+ < 0$ $\gamma_1^- > 0,\ \gamma_2^- < 0$
$S\,3$	curves with one inflection point	$S\,3.1$	$\beta_1^+ = \beta_2^+ = 0$ $\beta_1^- = \beta_2^- = 0$	$\gamma_1^+ > 0,\ \gamma_2^+ > 0$ $\gamma_1^- > 0,\ \gamma_2^- > 0$
		$S\,3.2$	$\beta_1^+ = \beta_2^+ = 0$ $\beta_1^- = \beta_2^- = 0$	$\gamma_1^+ > 0,\ \gamma_2^+ > 0$ $\gamma_1^- > 0,\ \gamma_2^- < 0$
		$S\,3.3$	$\beta_1^+ > 0,\ \beta_2^+ > 0$ $\beta_1^- < 0,\ \beta_2^- < 0$	$\gamma_1^+ > 0,\ \gamma_2^+ < 0$ $\gamma_1^- > 0,\ \gamma_2^- < 0$
$S\,4$	curves with two inflection points	–	$\beta_1^+ > 0,\ \beta_2^+ > 0$ $\beta_1^- < 0,\ \beta_2^- < 0$	$\gamma_1^+ > 0,\ \gamma_2^+ > 0$ $\gamma_1^- > 0,\ \gamma_2^- > 0$

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

conditions to be satisfied

$\alpha^{+} > 0 \quad \alpha^{-} > 0 \quad {f_0}^{+} > {f_0}^{-}$

${k_b}^{+} \quad {\beta_1}^{+} \quad {\beta_2}^{+} \quad \gamma^{+} \quad {\gamma_2}^{+} \quad {\gamma_3}^{+}$

${k_b}^{-} \quad {\beta_1}^{-} \quad {\beta_2}^{-} \quad \gamma^{-} \quad {\gamma_2}^{-} \quad {\gamma_3}^{-}$

can be arbitrary real numbers

shape type	limiting curves	subtype	obtained for	
$S\,1$	straight lines	–	${\beta_1}^{+} = {\beta_2}^{+} = 0$ ${\beta_1}^{-} = {\beta_2}^{-} = 0$	${\gamma_1}^{+} = {\gamma_2}^{+} = 0$ ${\gamma_1}^{-} = {\gamma_2}^{-} = 0$
$S\,2$	curves with no inflection point	$S\,2.1$	${\beta_1}^{+} > 0,\ {\beta_2}^{+} > 0$ ${\beta_1}^{-} > 0,\ {\beta_2}^{-} > 0$	${\gamma_1}^{+} = {\gamma_2}^{+} = 0$ ${\gamma_1}^{-} = {\gamma_2}^{-} = 0$
		$S\,2.2$	${\beta_1}^{+} > 0,\ {\beta_2}^{+} > 0$ ${\beta_1}^{-} < 0,\ {\beta_2}^{-} < 0$	${\gamma_1}^{+} = {\gamma_2}^{+} = 0$ ${\gamma_1}^{-} = {\gamma_2}^{-} = 0$
		$S\,2.3$	${\beta_1}^{+} > 0,\ {\beta_2}^{+} > 0$ ${\beta_1}^{-} < 0,\ {\beta_2}^{-} < 0$	${\gamma_1}^{+} > 0,\ {\gamma_2}^{+} < 0$ ${\gamma_1}^{-} > 0,\ {\gamma_2}^{-} < 0$
$S\,3$	curves with one inflection point	$S\,3.1$	${\beta_1}^{+} = {\beta_2}^{+} = 0$ ${\beta_1}^{-} = {\beta_2}^{-} = 0$	${\gamma_1}^{+} > 0,\ {\gamma_2}^{+} > 0$ ${\gamma_1}^{-} > 0,\ {\gamma_2}^{-} > 0$
		$S\,3.2$	${\beta_1}^{+} = {\beta_2}^{+} = 0$ ${\beta_1}^{-} = {\beta_2}^{-} = 0$	${\gamma_1}^{+} > 0,\ {\gamma_2}^{+} > 0$ ${\gamma_1}^{-} > 0,\ {\gamma_2}^{-} < 0$
		$S\,3.3$	${\beta_1}^{+} > 0,\ {\beta_2}^{+} > 0$ ${\beta_1}^{-} < 0,\ {\beta_2}^{-} < 0$	${\gamma_1}^{+} > 0,\ {\gamma_2}^{+} < 0$ ${\gamma_1}^{-} > 0,\ {\gamma_2}^{-} < 0$
$S\,4$	curves with two inflection points	–	${\beta_1}^{+} > 0,\ {\beta_2}^{+} > 0$ ${\beta_1}^{-} < 0,\ {\beta_2}^{-} < 0$	${\gamma_1}^{+} > 0,\ {\gamma_2}^{+} > 0$ ${\gamma_1}^{-} > 0,\ {\gamma_2}^{-} > 0$

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

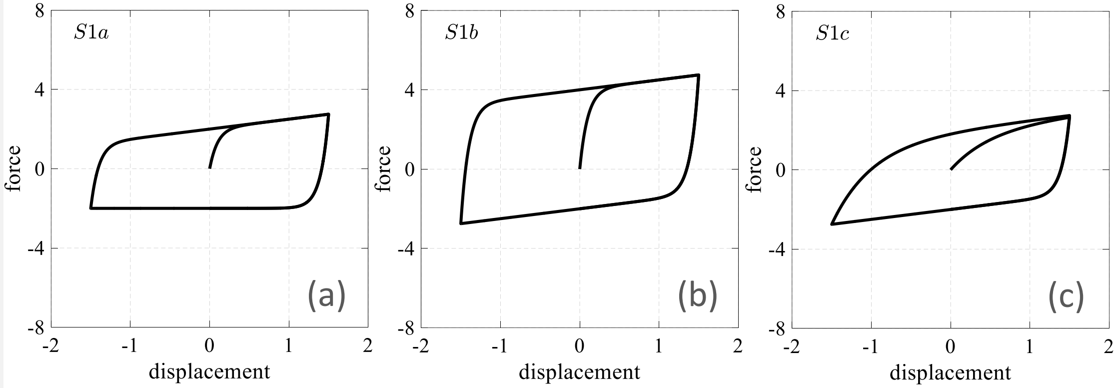
$k_b{}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



		k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
(a)	+	0.5	2	10	0	0	0	0	0
	–	0	2	10	0	0	0	0	0
(b)	+	0.5	4	10	0	0	0	0	0
	–	0.5	2	10	0	0	0	0	0
(c)	+	0.5	2	2	0	0	0	0	0
	–	0.5	2	10	0	0	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

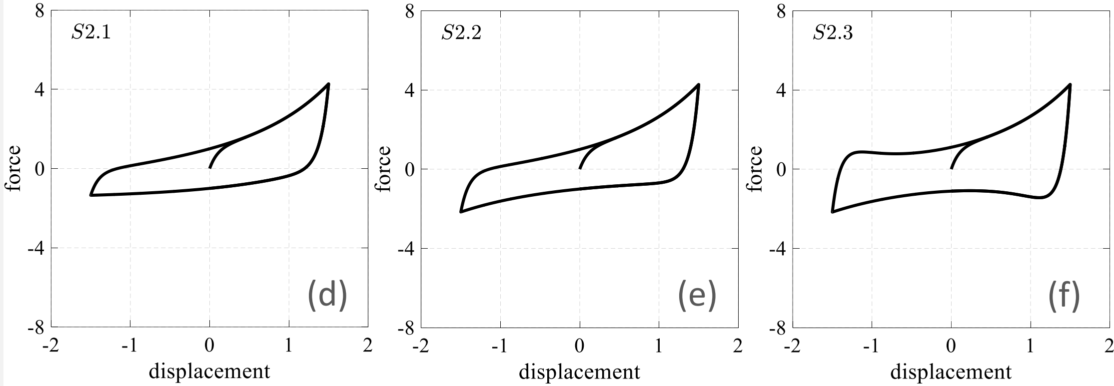
$k_b{}^+ \quad f_0{}^+ \quad \alpha^+$

$\beta_1{}^+ \quad \beta_2{}^+ \quad \gamma_1{}^+ \quad \gamma_2{}^+ \quad \gamma_3{}^+$

unloading phase

$k_b{}^- \quad f_0{}^- \quad \alpha^-$

$\beta_1{}^- \quad \beta_2{}^- \quad \gamma^- \quad \gamma_2{}^- \quad \gamma_3{}^-$



		k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
(d)	+	0.5	1	10	0.5	1.2	0	0	0
	−	0	1	10	0.5	0.8	0	0	0
(e)	+	0.5	1	10	0.5	1.2	0	0	0
	−	0	1	10	-0.5	-0.8	0	0	0
(f)	+	0.5	4	10	0.5	1.2	1.5	-2	-2
	−	0	4	10	-0.5	-0.8	1.5	-2	2

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

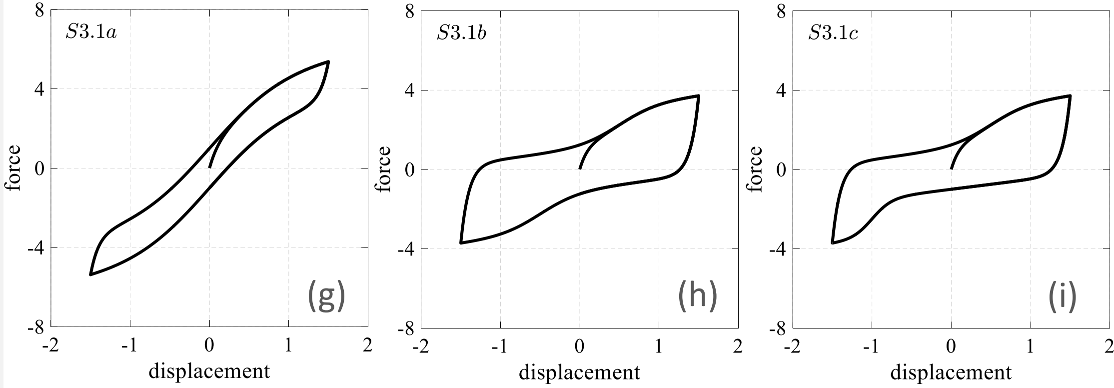
$k_b{}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



		k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
(g)	+	0.5	1	10	0	0	2	2	0
	–	0.5	1	10	0	0	2	2	0
(h)	+	0.5	2	10	0	0	0.5	4	0.5
	–	0.5	2	10	0	0	0.5	4	-0.5
(i)	+	0.5	2	10	0	0	0.5	4	0.5
	–	0.5	2	10	0	0	0.5	8	-1

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

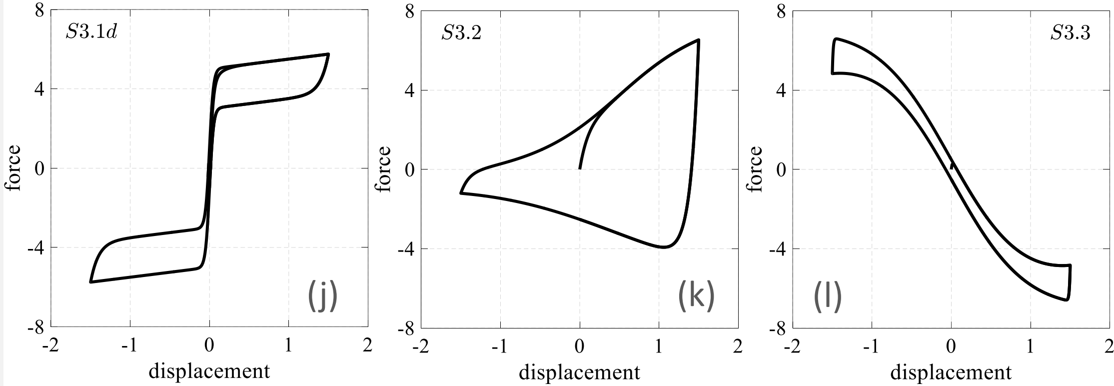
$k_b{}^+ \quad f_0{}^+ \quad \alpha^+$

$\beta_1{}^+ \quad \beta_2{}^+ \quad \gamma_1{}^+ \quad \gamma_2{}^+ \quad \gamma_3{}^+$

unloading phase

$k_b{}^- \quad f_0{}^- \quad \alpha^-$

$\beta_1{}^- \quad \beta_2{}^- \quad \gamma^- \quad \gamma_2{}^- \quad \gamma_3{}^-$



		k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
(j)	+	0.5	1	10	0	0	2	40	0
	–	0.5	1	10	0	0	2	40	0
(k)	+	0.5	3.5	10	0	0	1.5	2	0.5
	–	0.5	3.5	10	0	0	2	-1	0.5
(l)	+	0.5	0.5	100	0.5	0.8	4	-2	0
	–	0.5	0.5	100	-0.5	-0.8	4	-2	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

model parameters

loading phase

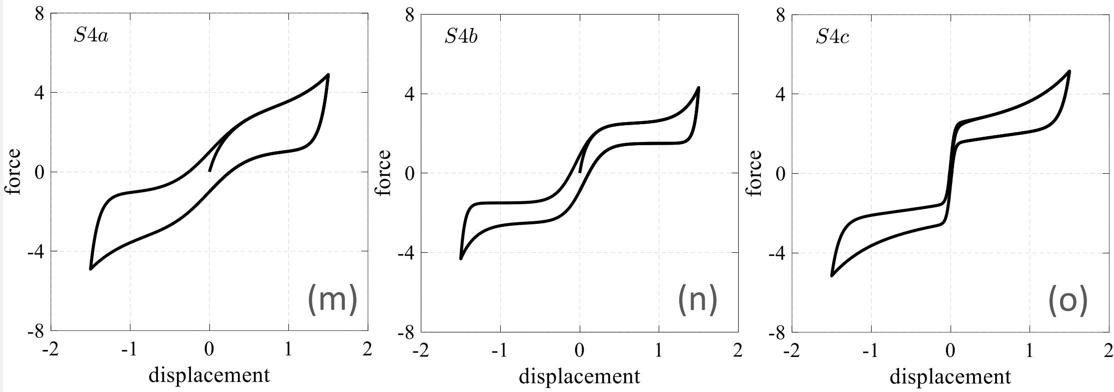
$k_b{}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



		k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
(m)	+	0	1	10	0.1	2	1	4	0
	–	0	1	10	-0.1	-2	1	4	0
(n)	+	0	0.5	20	0.001	5	1	8	-0.05
	–	0	0.5	20	-0.001	-5	1	8	0.05
(o)	+	0.5	0.5	10	0.1	2	1	40	0
	–	0.5	0.5	10	-0.1	-2	1	40	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

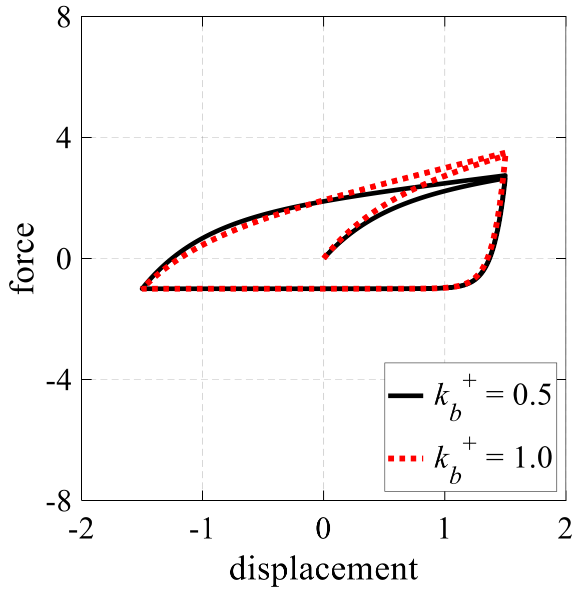
$\boldsymbol{k_b}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	2	2	0	0	0	0	0
−	0	1	10	0	0	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

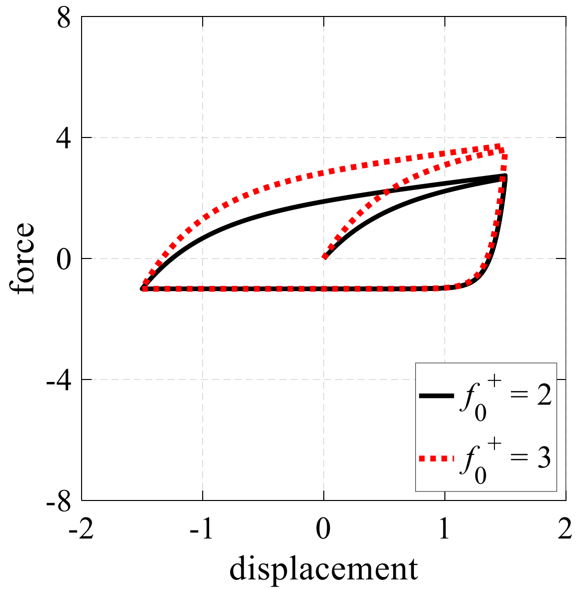
$k_b{}^+$ $\boldsymbol{f_0{}^+}$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	2	2	0	0	0	0	0
−	0	1	10	0	0	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

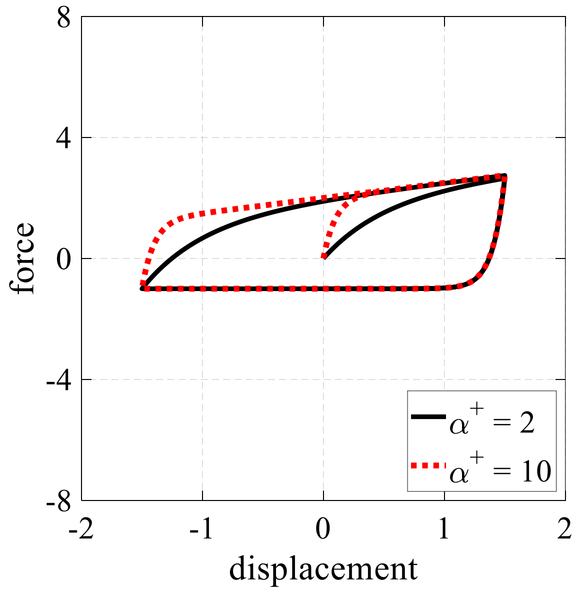
$k_b{}^+$ $f_0{}^+$ $\boldsymbol{\alpha}^+$

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	2	2	0	0	0	0	0
−	0	1	10	0	0	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

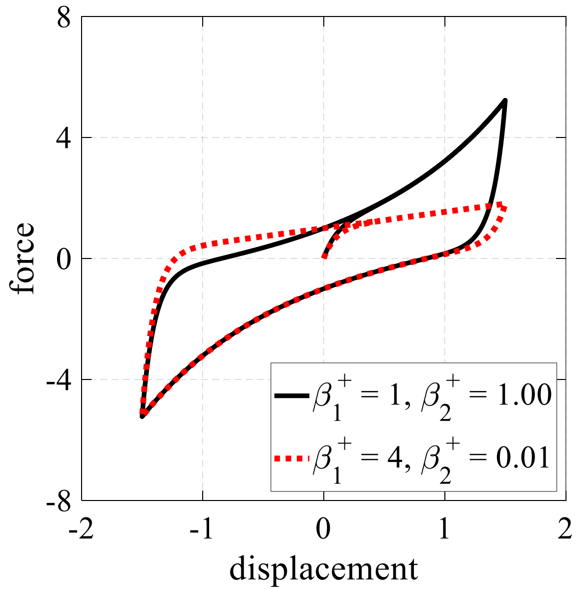
$k_b{}^+$ $f_0{}^+$ α^+

$\boldsymbol{\beta_1}^+$ $\boldsymbol{\beta_2}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	1	10	1	1	0	0	0
−	0.5	1	10	-1	-1	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

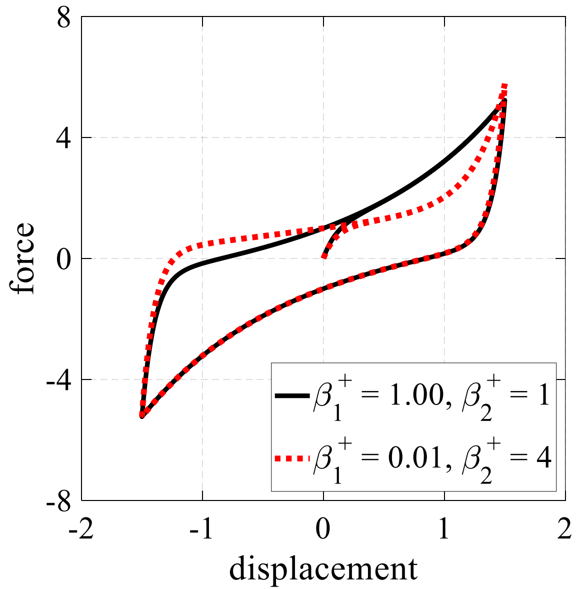
$k_b{}^+$ $f_0{}^+$ α^+

$\boldsymbol{\beta_1}^+$ $\boldsymbol{\beta_2}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	1	10	1	1	0	0	0
−	0.5	1	10	-1	-1	0	0	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

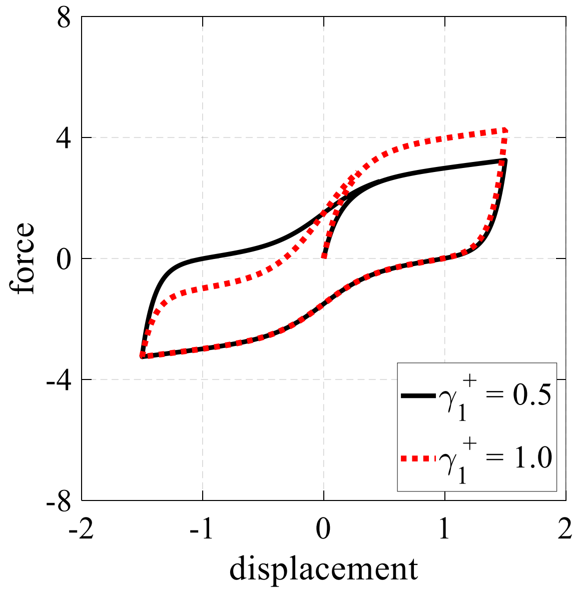
$k_b{}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\boldsymbol{\gamma_1}{}^+$ $\gamma_2{}^+$ $\gamma_3{}^+$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	1.5	10	0	0	0.5	5	0
–	0.5	1.5	10	0	0	0.5	5	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

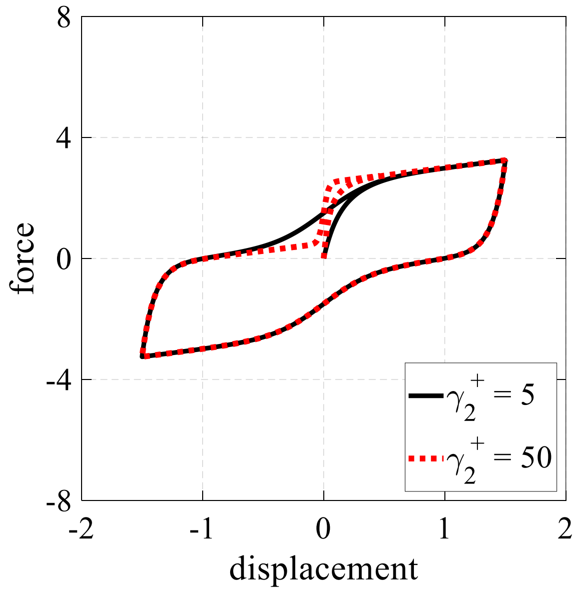
k_b^+ f_0^+ α^+

β_1^+ β_2^+ γ_1^+ $\boldsymbol{\gamma_2^+}$ γ_3^+

unloading phase

k_b^- f_0^- α^-

β_1^- β_2^- γ^- γ_2^- γ_3^-



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	1.5	10	0	0	0.5	5	0
−	0.5	1.5	10	0	0	0.5	5	0

Modeling of complex hysteresis loops

Vaiana-Rosati model

parameter sensitivity analysis

loading phase

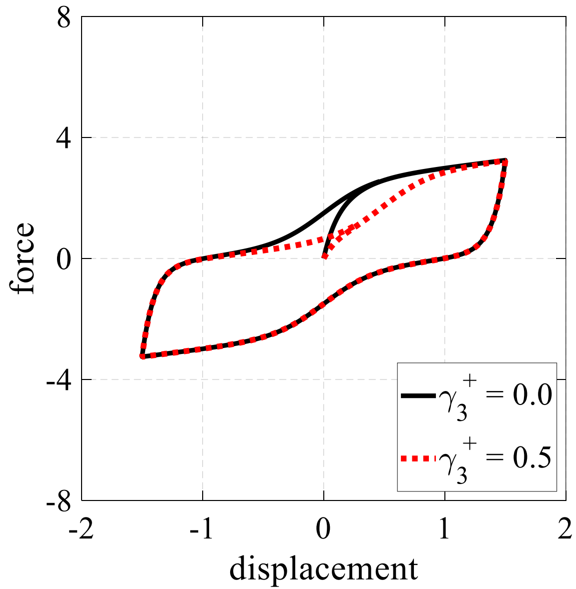
$k_b{}^+$ $f_0{}^+$ α^+

$\beta_1{}^+$ $\beta_2{}^+$ $\gamma_1{}^+$ $\gamma_2{}^+$ $\boldsymbol{\gamma_3{}^+}$

unloading phase

$k_b{}^-$ $f_0{}^-$ α^-

$\beta_1{}^-$ $\beta_2{}^-$ γ^- $\gamma_2{}^-$ $\gamma_3{}^-$



	k_b	f_0	α	β_1	β_2	γ_1	γ_2	γ_3
+	0.5	1.5	10	0	0	0.5	5	0
−	0.5	1.5	10	0	0	0.5	5	0

Validation of the Vaiana-Rosati model

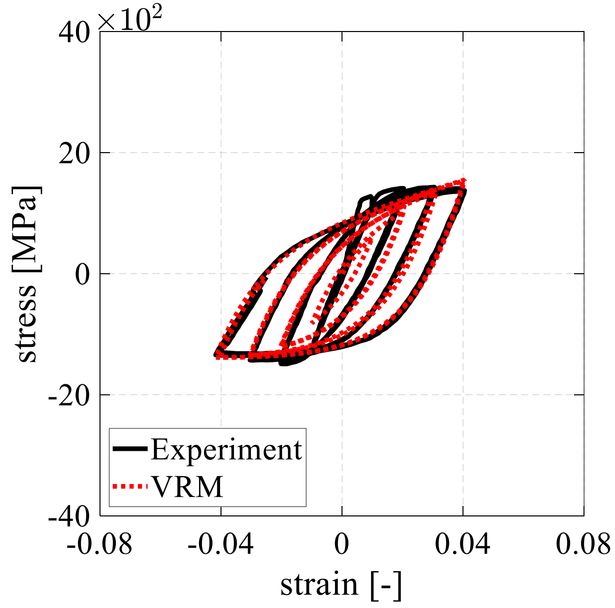
Validation against experimental results

steel bar
(Han et al. 2019)

steel damper
(Zhai et al. 2020)

negative stiffness device
(Sarlis et al. 2013)

SMA assembly
(Dolce and Cardone 2001)



	k_b [MPa]	f_0 [MPa]	α [−]	β_1 [MPa]	β_2 [−]	γ_1 [MPa]	γ_2 [−]	γ_3 [−]
+	15000	950	65	0	0	0	0	0
−	−1000	1450	60	0	0	0	0	0

Validation of the Vaiana-Rosati model

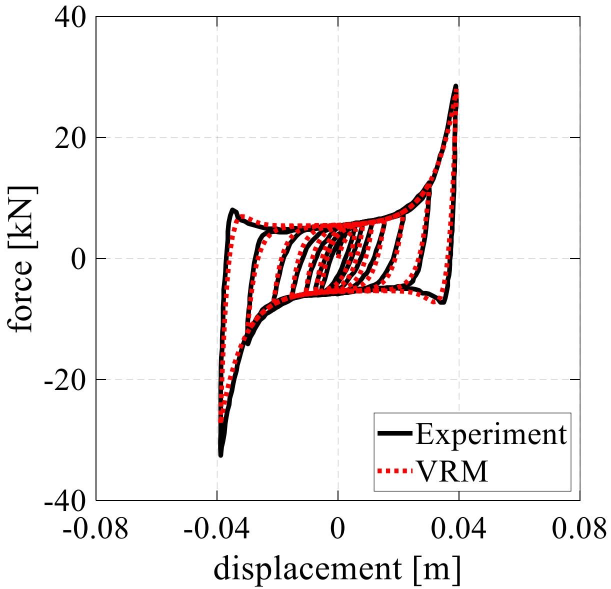
Validation against experimental results

steel bar
(Han et al. 2019)

steel damper
(Zhai et al. 2020)

negative stiffness device
(Sarlis et al. 2013)

SMA assembly
(Dolce and Cardone 2001)



	k_b [kNm^{-1}]	f_0 [kN]	α [m^{-1}]	β_1 [kN]	β_2 [m^{-1}]	γ_1 [kN]	γ_2 [m^{-1}]	γ_3 [m]
+	0	15.5	330	0.14	130	5	−350	−0.034
−	0	15.5	330	−0.14	−130	5	−350	0.034

Validation of the Vaiana-Rosati model

Validation against experimental results

steel bar
(Han et al. 2019)

steel damper
(Zhai et al. 2020)

negative stiffness device
(Sarlis et al. 2013)

SMA assembly
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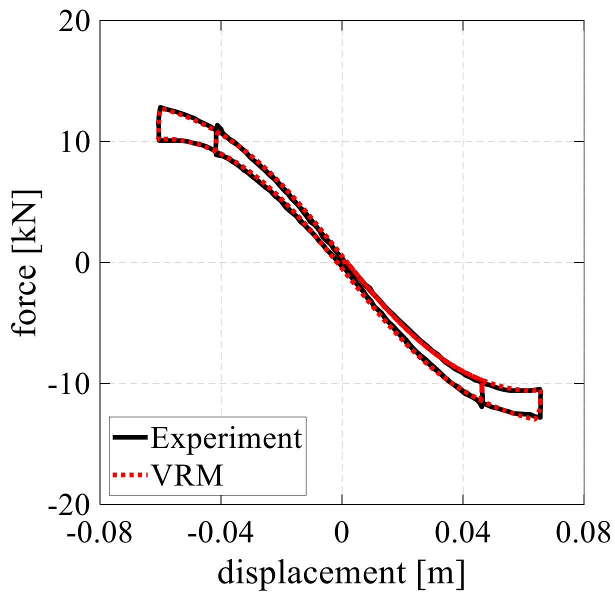


	k_b [kNm^{-1}]	f_0 [kN]	α [m^{-1}]	β_1 [kN]	β_2 [m^{-1}]	γ_1 [kN]	γ_2 [m^{-1}]	γ_3 [m]
+	12	1	5000	0.001	100	7.2	−45	−0.0015
−	12	1	5000	−0.002	−100	7.2	−45	0.0015

Validation of the Vaiana-Rosati model

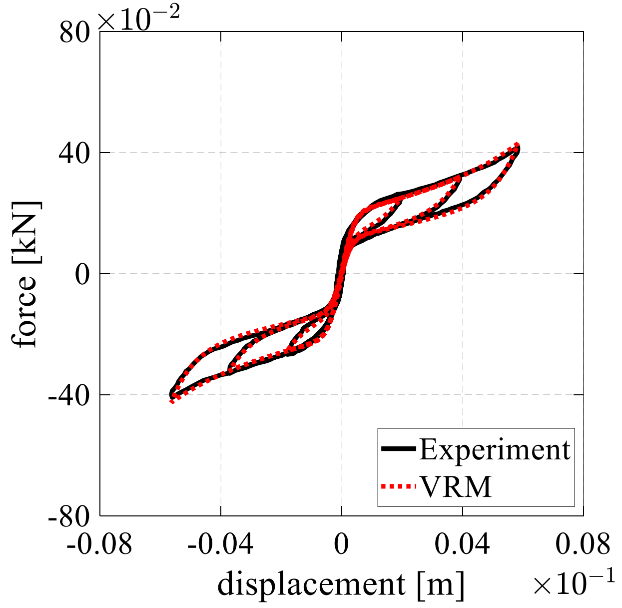
Validation against experimental results

steel bar
(Han et al. 2019)

steel damper
(Zhai et al. 2020)

negative stiffness device
(Sarlis et al. 2013)

SMA assembly
(Dolce and Cardone 2001)



	k_b [kNm^{-1}]	f_0 [kN]	α [m^{-1}]	β_1 [kN]	β_2 [m^{-1}]	γ_1 [kN]	γ_2 [m^{-1}]	γ_3 [m]
+	20	0.038	1300	0.009	450	0.08	5400	0.00005
−	20	0.048	1300	−0.009	−450	0.08	5400	−0.0001

Validation of the Vaiana-Rosati model

Validation against numerical results

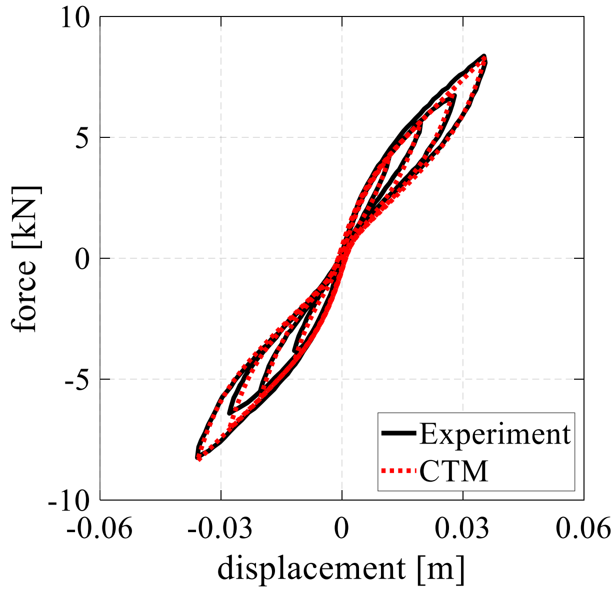
SDOF hysteretic mechanical system

$m\ddot{u} + f = p(t)$

Ni-Ti SMA helical spring
(Zhuang et al. 2016)

Charalampakis and Tsiatas Model (CTM)
(Charalampakis and Tsiatas 2018)

differential model



k_1 [kNm^{-1}]	k_2 [kNm^{-1}]	$\bar{f}$ [kN]	n [−]	a_1 [m^{-1}]	a_2 [m]	b_1 [−]	b_2 [m]	b_3 [m^{-1}]
580	0	4.2	1.45	0	0	0.395	0.0173	450

Validation of the Vaiana-Rosati model

Validation against numerical results

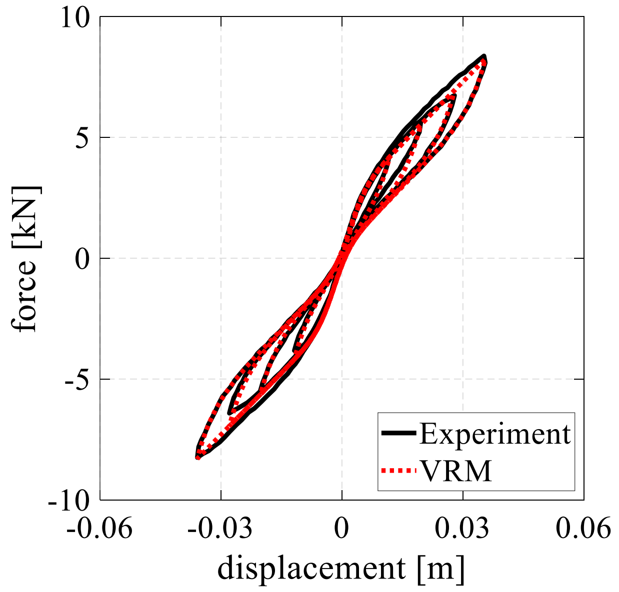
SDOF hysteretic mechanical system

$m\ddot{u} + f = p(t)$

Ni-Ti SMA helical spring
(Zhuang et al. 2016)

Vaiana and Rosati Model (VRM)
(Vaiana and Rosati 2023)

exponential model



	k_b [kNm^{-1}]	f_0 [kN]	α [m^{-1}]	β_1 [kN]	β_2 [m^{-1}]	γ_1 [kN]	γ_2 [m^{-1}]	γ_3 [m]
+	170	0.9	265	0	0	0.65	500	0.002
−	170	0.9	265	0	0	0.65	500	- 0.002

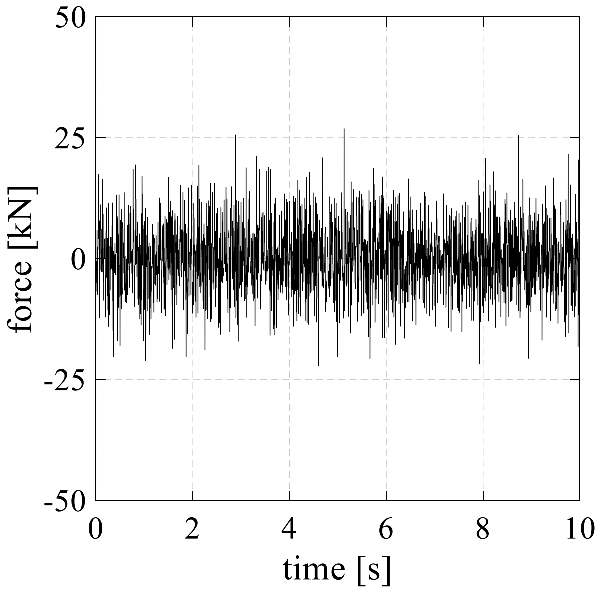
Validation of the Vaiana-Rosati model

Validation against numerical results

SDOF hysteretic mechanical system

$$m\ddot{u} + f = p(t)$$

applied external random force



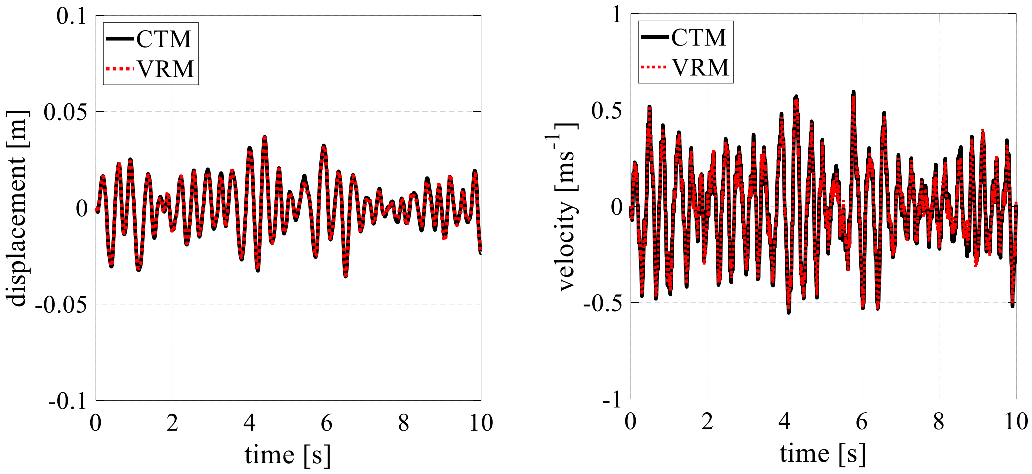
Validation of the Vaiana-Rosati model

Validation against numerical results

SDOF hysteretic mechanical system

$$m\ddot{u} + f = p(t)$$

NLTHAs results



$$\text{VRM } tctp\ [\%] = \frac{\text{VRM } tct}{\text{CTM } tct}\ 100$$

	$tct\ [s]$	$tctp$
CTM	44.32	-
VRM	0.54	**1.21 %**

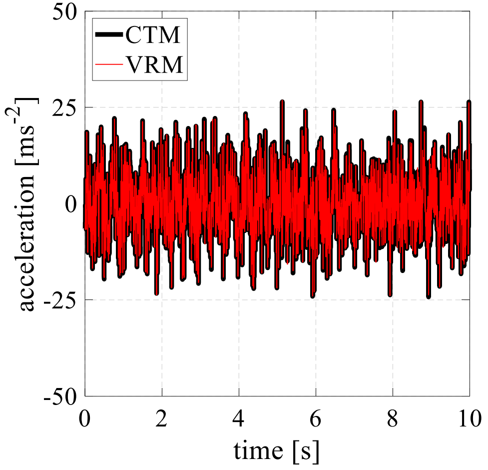
Validation of the Vaiana-Rosati model

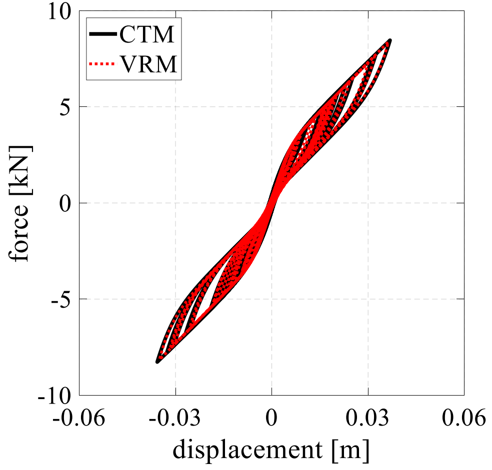
Validation against numerical results

SDOF hysteretic mechanical system

$$m\ddot{u} + f = p(t)$$

NLTHAs results





$$\text{VRM } tctp\ [\%] = \frac{\text{VRM } tct}{\text{CTM } tct}\ 100$$

	$tct\ [s]$	$tctp$
CTM	44.32	-
VRM	0.54	**1.21 %**

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

Advantages

A generic hysteresis loop is described by only two curves

No internal variables need to be evaluated

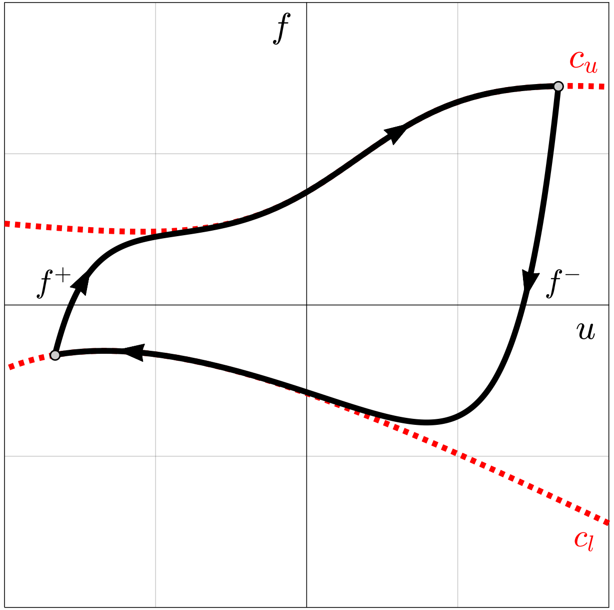
Closed form expressions can be faster implemented

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$f^+(u, u_P, f_P) = {f_e}^+(u) + {k_b}^+ u + {f_0}^+ - \left({f_e}^+(u_P) + {k_b}^+ u_P + {f_0}^+ - f_P\right) \times e^{-\alpha^+(u-u_P)}$$

$${f_e}^+(u) = {\beta_1}^+ e^{{\beta_2}^+ u} - {\beta_1}^+ + \frac{4\,{\gamma_1}^+}{1 + e^{-{\gamma_2}^+(u-{\gamma_3}^+)}} - 2{\gamma_1}^+$$



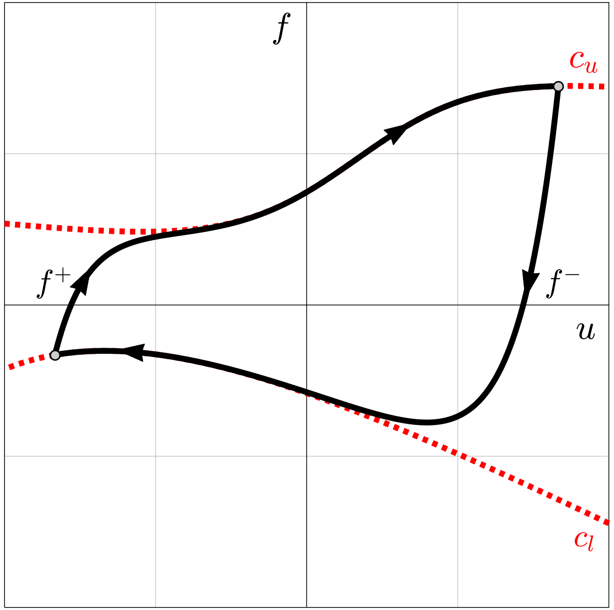
generalized force

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$f^+(u, u_P, f_P) = f_e{}^+(u) + k_b{}^+u + f_0{}^+ - \left(f_e{}^+(u_P) + k_b{}^+u_P + f_0{}^+ - f_P\right) \times e^{-\alpha^+(u-u_P)}$$

$$f_e{}^+(u) = \beta_1{}^+ e^{\beta_2{}^+ u} - \beta_1{}^+ + \frac{4\,\gamma_1{}^+}{1 + e^{-\gamma_2{}^+(u-\gamma_3{}^+)}} - 2\gamma_1{}^+$$



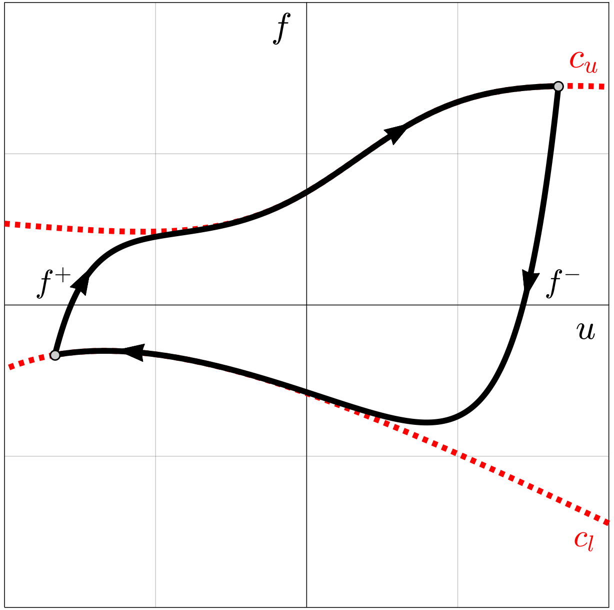
generalized force

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$f^-(u, u_P, f_P) = f_e{}^-(u) + k_b{}^- u - f_0{}^- - \left(f_e{}^-(u_P) + k_b{}^- u_P - f_0{}^- - f_P\right) \times e^{+\alpha^-(u-u_P)}$$

$$f_e{}^-(u) = \beta_1{}^- e^{\beta_2{}^- u} - \beta_1{}^- + \frac{4\,\gamma_1{}^-}{1 + e^{-\gamma_2{}^-(u-\gamma_3{}^-)}} - 2\gamma_1{}^-$$



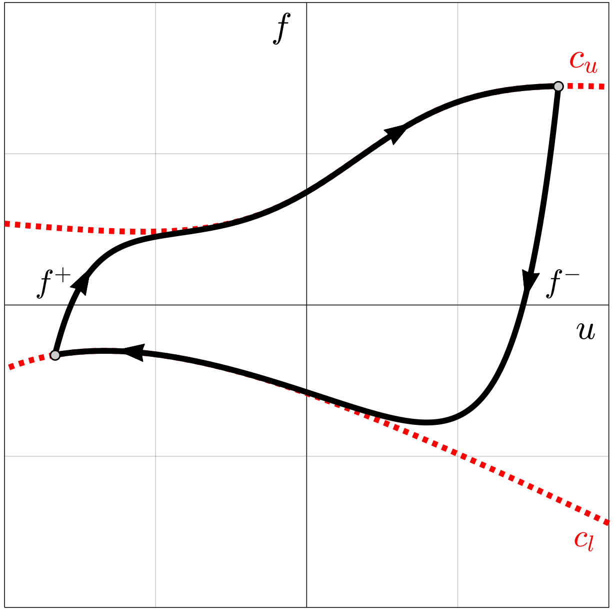
generalized force

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$f^-(u, u_P, f_P) = f_e{}^-(u) + k_b{}^- u - f_0{}^- - (f_e{}^-(u_P) + k_b{}^- u_P - f_0{}^- - f_P) \times e^{+\alpha^-(u-u_P)}$$

$$f_e{}^-(u) = \beta_1{}^- e^{\beta_2{}^- u} - \beta_1{}^- + \frac{4\,\gamma_1{}^-}{1 + e^{-\gamma_2{}^-(u-\gamma_3{}^-)}} - 2\gamma_1{}^-$$



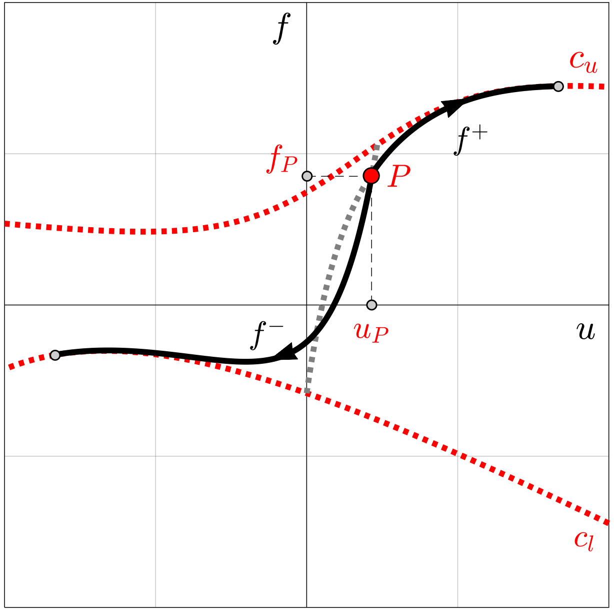
generalized force

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$f^+(u, u_P, f_P) = f_e{}^+(u) + k_b{}^+ u + f_0{}^+ - \left(f_e{}^+(u_P) + k_b{}^+ u_P + f_0{}^+ - f_P\right) \times e^{-\alpha^+(u - u_P)}$$

$$f^-(u, u_P, f_P) = f_e{}^-(u) + k_b{}^- u - f_0{}^- - \left(f_e{}^-(u_P) + k_b{}^- u_P - f_0{}^- - f_P\right) \times e^{+\alpha^-(u - u_P)}$$



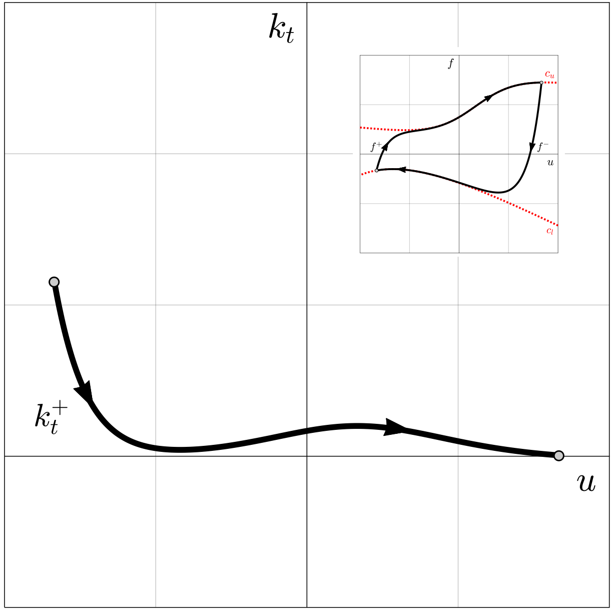
generalized force

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$k_t^{+}(u, u_P, f_P) = k_e^{+}(u) + k_b^{+} + \left(f_e^{+}(u_P) + k_b^{+} u_P + f_0^{+} - f_P\right) \times \alpha^{+} e^{-\alpha^{+}(u-u_P)}$$

$$k_e^{+}(u) = \beta_1^{+} \beta_2^{+} e^{\beta_2^{+} u} + \frac{4\,\gamma_1^{+} \gamma_2^{+} e^{-\gamma_2^{+}(u-\gamma_3^{+})}}{\left[1 + e^{-\gamma_2^{+}(u-\gamma_3^{+})}\right]^2}$$



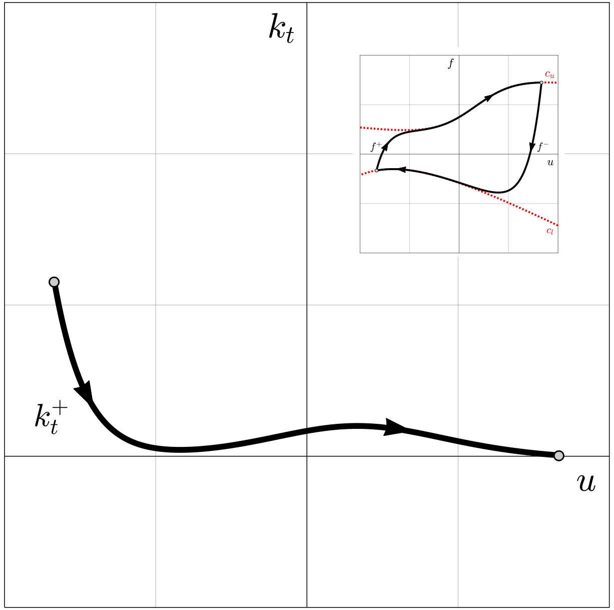
generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$k_t^{+}(u,u_P,f_P) = k_e^{+}(u) + k_b^{+} + \left(f_e^{+}(u_P) + k_b^{+}u_P + f_0^{+} - f_P\right) \times \alpha^{+}e^{-\alpha^{+}(u-u_P)}$$

$$k_e^{+}(u) = \beta_1^{+}\beta_2^{+}e^{\beta_2^{+}u} + \frac{4\,\gamma_1^{+}\gamma_2^{+}e^{-\gamma_2^{+}(u-\gamma_3^{+})}}{\left[1+e^{-\gamma_2^{+}(u-\gamma_3^{+})}\right]^2}$$



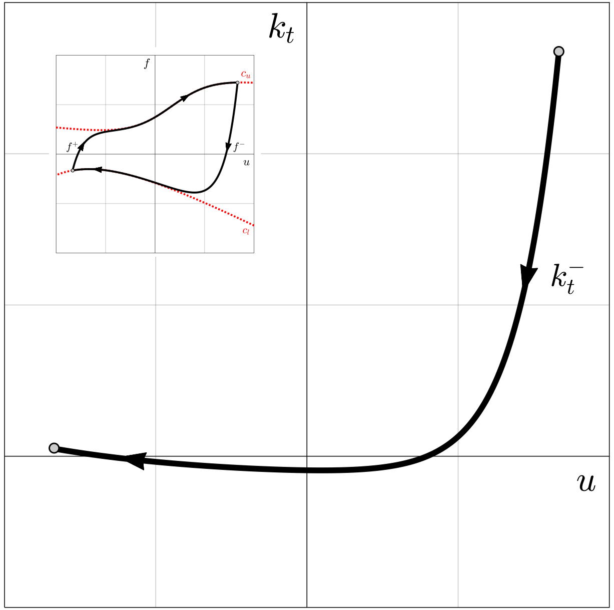
generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$k_t^-(u, u_P, f_P) = k_e^-(u) + k_b^- - (f_e^-(u_P) + k_b^- u_P - f_0^- - f_P) \times \alpha^- e^{+\alpha^-(u-u_P)}$$

$$k_e^-(u) = \beta_1^- \beta_2^- e^{\beta_2^- u} + \frac{4\,\gamma_1^- \gamma_2^- e^{-\gamma_2^-(u-\gamma_3^-)}}{[1+e^{-\gamma_2^-(u-\gamma_3^-)}]^2}$$



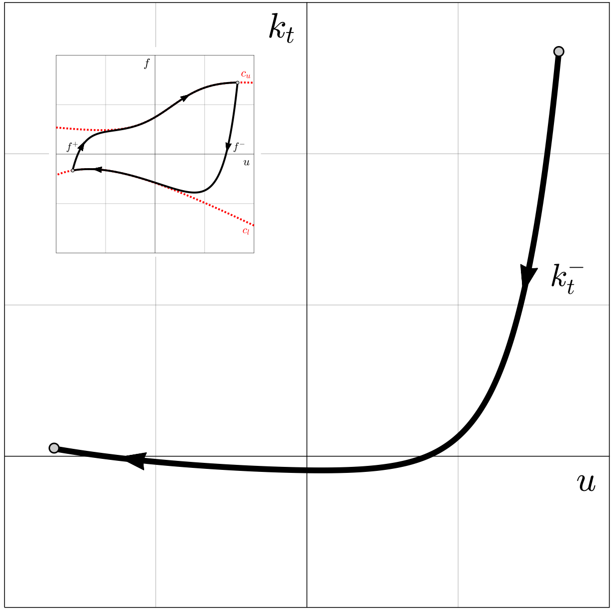
generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$k_t{}^-(u, u_P, f_P) = k_e{}^-(u) + k_b{}^- - \left(f_e{}^-(u_P) + k_b{}^- u_P - f_0{}^- - f_P\right) \times \alpha^- e^{+\alpha^-(u-u_P)}$$

$$k_e{}^-(u) = \beta_1{}^- \beta_2{}^- e^{\beta_2{}^- u} + \frac{4\,\gamma_1{}^- \gamma_2{}^- e^{-\gamma_2{}^-(u-\gamma_3{}^-)}}{[1 + e^{-\gamma_2{}^-(u-\gamma_3{}^-)}]^2}$$

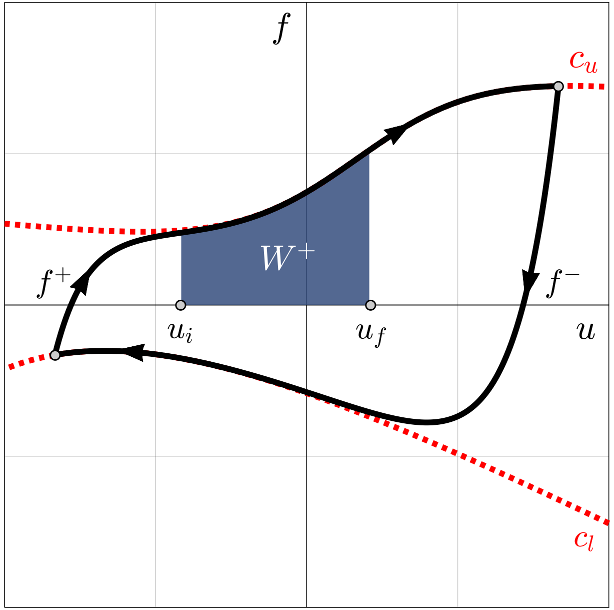


generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$W^{+}(u_i, u_f, u_P, f_P) = {W_a}^{+}(u_i, u_f) + {W_b}^{+}(u_i, u_f) + {W_c}^{+}(u_i, u_f) + {W_d}^{+}(u_i, u_f, u_P, f_P)$$

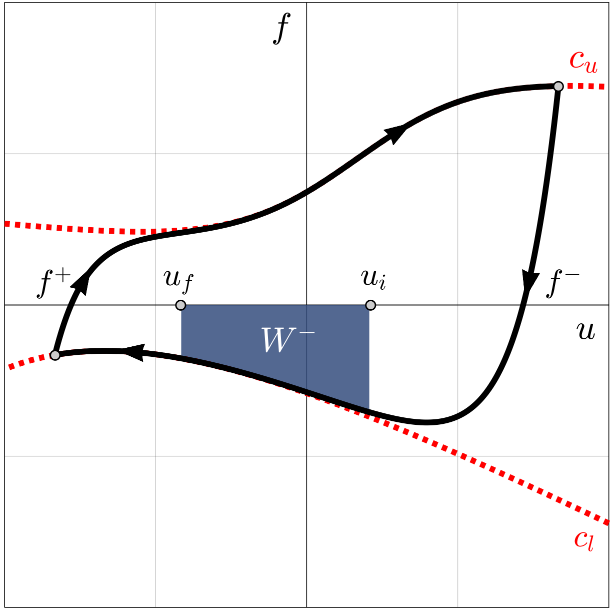


generalized work

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

$$
\begin{aligned}
W^-(u_i, u_f, u_P, f_P) = &\ W_a{}^-(u_i, u_f) \\
&+ W_b{}^-(u_i, u_f) \\
&+ W_c{}^-(u_i, u_f) \\
&+ W_d{}^-(u_i, u_f, u_P, f_P)
\end{aligned}
$$



generalized work

Reformulation of the Vaiana-Rosati model

Analytical reformulation (VRM+A)

implementation algorithm

1 Initial settings

1.1 Set the model parameters

$k_b^+, f_0^+, \alpha^+, \beta_1^+, \beta_2^+, \gamma_1^+, \gamma_2^+, \gamma_3^+$ and $k_b^-, f_0^-, \alpha^-, \beta_1^-, \beta_2^-, \gamma_1^-, \gamma_2^-, \gamma_3^-$

1.2 Define initial values of generalized force, tangent stiffness, and work

$f_{t=0}\,, (k_t)_{t=0}\,, W_{t=0}$

2 Calculations at each time step

2.1 Update the model parameters

$k_b = k_b^+\,(k_b^-),\ f_0 = f_0^+\,(f_0^-),\ \alpha = \alpha^+\,(\alpha^-),\ \beta_1 = \beta_1^+\,(\beta_1^-),\ \beta_2 = \beta_2^+\,(\beta_2^-),$
$\gamma_1 = \gamma_1^+\,(\gamma_1^-),\ \gamma_2 = \gamma_2^+\,(\gamma_2^-),\ \gamma_3 = \gamma_3^+\,(\gamma_3^-)$, if $s_t > 0\ (s_t < 0)$

2.2 Evaluate the generalized force at time t

$$(f_e)_{t-\Delta t} = \beta_1 e^{\beta_2 u_{t-\Delta t}} - \beta_1 + \frac{4\gamma_1}{1 + e^{-\gamma_2(u_{t-\Delta t} - \gamma_3)}} - 2\gamma_1$$

$$(f_e)_t = \beta_1 e^{\beta_2 u_t} - \beta_1 + \frac{4\gamma_1}{1 + e^{-\gamma_2(u_t - \gamma_3)}} - 2\gamma_1$$

$$f_t = (f_e)_t + k_b\, u_t + s_t\, f_0 - [(f_e)_{t-\Delta t} + k_b\, u_{t-\Delta t} + s_t\, f_0 - f_{t-\Delta t}]e^{-s_t \alpha (u_t - u_{t-\Delta t})}$$

2.3 Compute the generalized tangent stiffness at time t

2.4 Calculate the generalized work at time t

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

Advantages

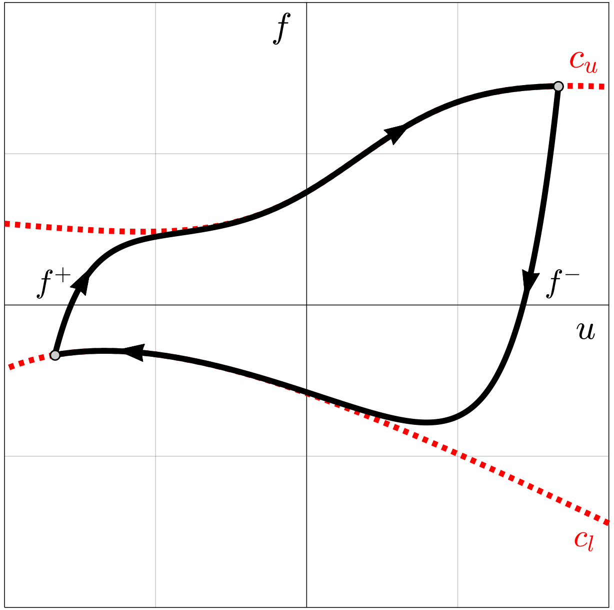
Adoption in nonlinear dynamics (state space formulation)

Extension to multiaxial cases

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$\dot{f}^+ = \Big[{k_e}^+(u) + {k_b}^+$$
$$+\alpha^+\big({f_e}^+(u) + {k_b}^+ u + {f_0}^+ - f^+\big)\Big]\,\dot{u}$$

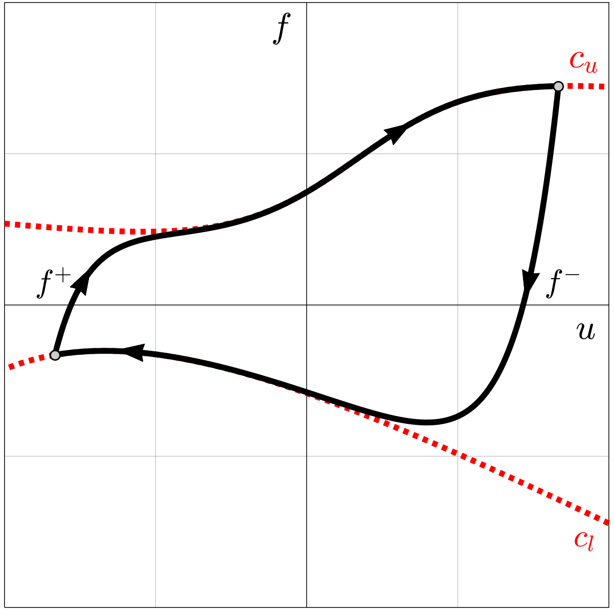


generalized force

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$\dot{f}^- = \left[k_e{}^-(u) + k_b{}^- \right. \\ \left. -\alpha^-(f_e{}^-(u) + k_b{}^- u - f_0{}^- - f^-)\right] \dot{u}$$

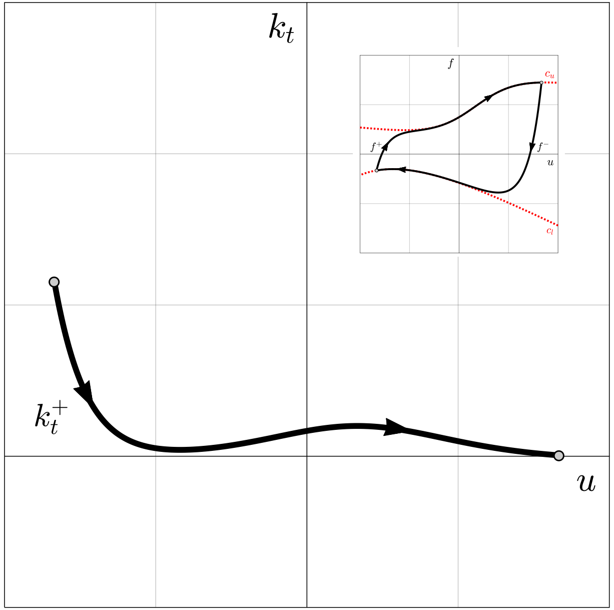


generalized force

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$k_t{}^+ = k_e{}^+(u) + k_b{}^+$$
$$+\alpha^+\left(f_e{}^+(u) + k_b{}^+ u + f_0{}^+ - f^+\right)$$

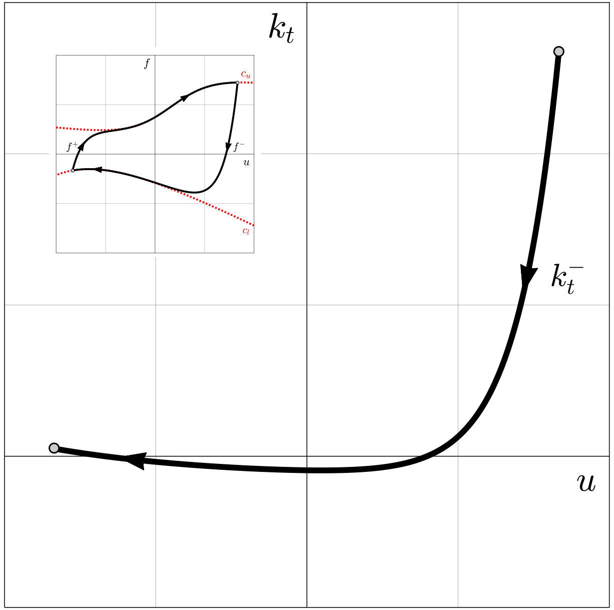


generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$k_t^- = k_e^-(u) + k_b^-$$
$$-\alpha^-(f_e^-(u) + k_b^- u - f_0^- - f^-)$$

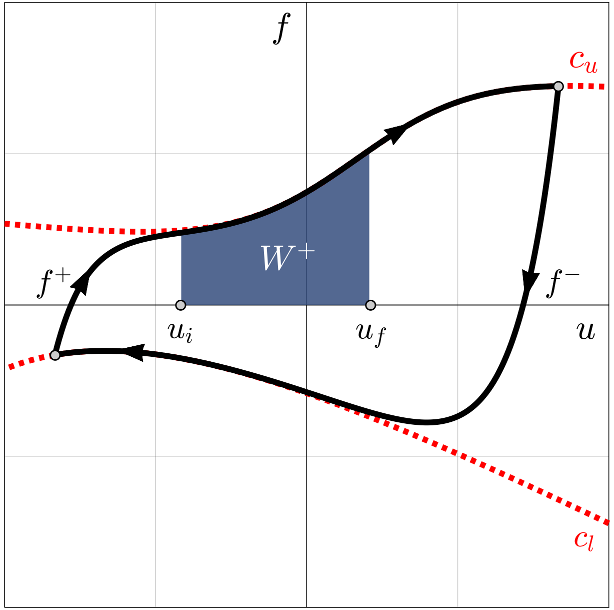


generalized tangent stiffness

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$\dot{W}^+ = f^+ \, \dot{u}$$

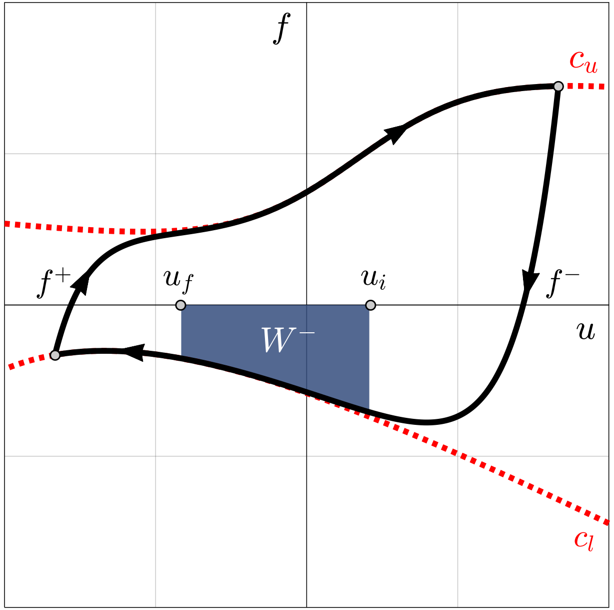


generalized work

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

$$\dot{W}^- = f^- \, \dot{u}$$



generalized work

Reformulation of the Vaiana-Rosati model

Differential reformulation (VRM+D)

implementation algorithm

1 Initial settings

1.1 Set the model parameters

$k_b^+, f_0^+, \alpha^+, \beta_1^+, \beta_2^+, \gamma_1^+, \gamma_2^+, \gamma_3^+$ and $k_b^-, f_0^-, \alpha^-, \beta_1^-, \beta_2^-, \gamma_1^-, \gamma_2^-, \gamma_3^-$

1.2 Define initial values of generalized force, tangent stiffness, and work

$f_{t=0}, (k_t)_{t=0}, W_{t=0}$

2 Calculations at each time step

2.1 Update the model parameters

$k_b = k_b^+ \, (k_b^-),\ f_0 = f_0^+ \, (f_0^-),\ \alpha = \alpha^+ \, (\alpha^-),\ \beta_1 = \beta_1^+ \, (\beta_1^-),\ \beta_2 = \beta_2^+ \, (\beta_2^-),$
$\gamma_1 = \gamma_1^+ \, (\gamma_1^-),\ \gamma_2 = \gamma_2^+ \, (\gamma_2^-),\ \gamma_3 = \gamma_3^+ \, (\gamma_3^-)$, if $s_t > 0 \ (s_t < 0)$

2.2 Evaluate the generalized force at time t by using a numerical method

$$(k_e)_t = \beta_1 \beta_2 e^{\beta_2 u_t} + \frac{4\gamma_1 \gamma_2 e^{-\gamma_2 (u_t - \gamma_3)}}{[1 + e^{-\gamma_2 (u_t - \gamma_3)}]^2}$$

$$(f_e)_t = \beta_1 e^{\beta_2 u_t} - \beta_1 + \frac{4\gamma_1}{1 + e^{-\gamma_2 (u_t - \gamma_3)}} - 2\gamma_1$$

$$\dot{f}_t = \left[(k_e)_t + k_b + s_t \alpha \left((f_e)_t + k_b\, u_t + s_t\, f_0 - f_t\right)\right] \dot{u}_t$$

2.3 Compute the generalized tangent stiffness at time t

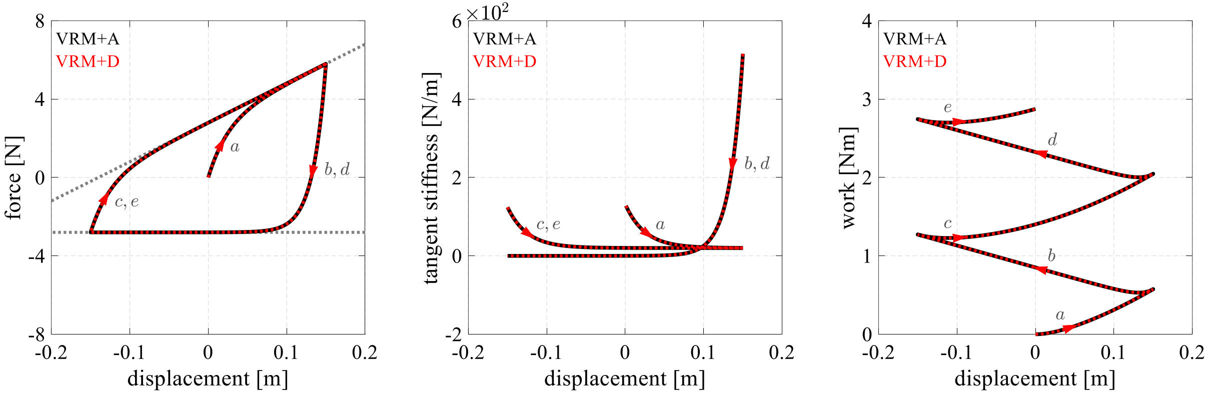
2.4 Calculate the generalized work at time t by using a numerical method

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S1

$$m\ddot{u} + f = p(t)$$



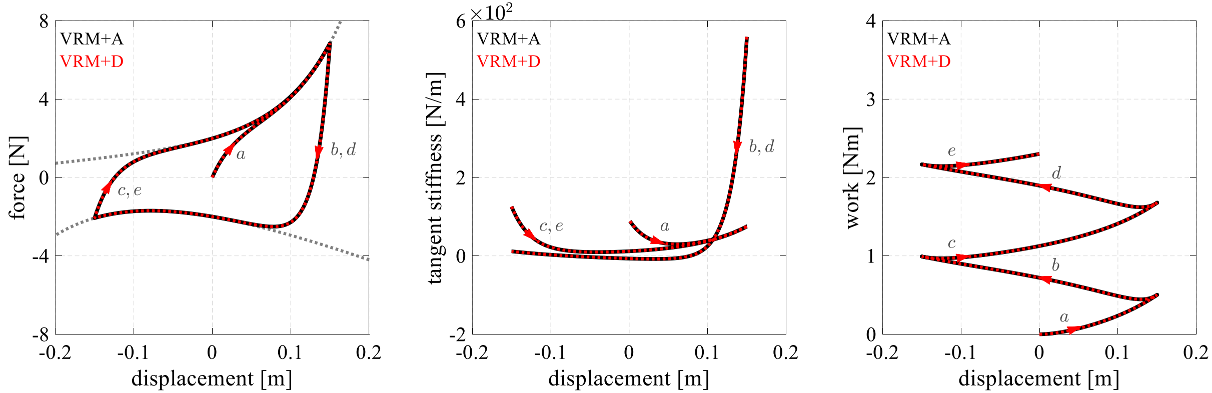
	k_b [Nm^{-1}]	f_0 [N]	α [m^{-1}]	β_1 [N]	β_2 [m^{-1}]	γ_1 [N]	γ_2 [m^{-1}]	γ_3 [m]
+	20	2.8	40	0	0	0	0	0
−	0	2.8	60	0	0	0	0	0

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S2

$m\ddot{u} + f = p(t)$



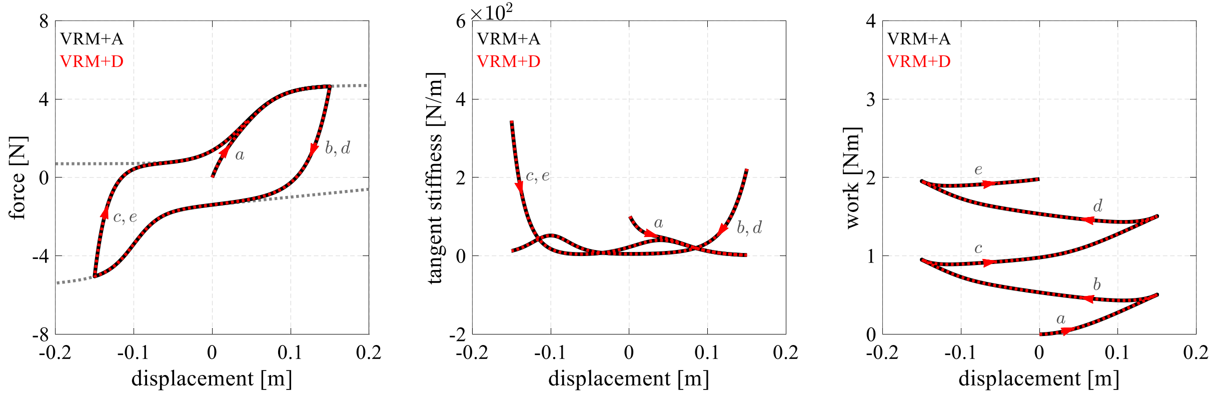
	k_b [Nm^{-1}]	f_0 [N]	α [m^{-1}]	β_1 [N]	β_2 [m^{-1}]	γ_1 [N]	γ_2 [m^{-1}]	γ_3 [m]
+	4	2	40	0.5	15	0	0	0
−	-15	2	55	-1	-8	0	0	0

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S3

$m\ddot{u} + f = p(t)$



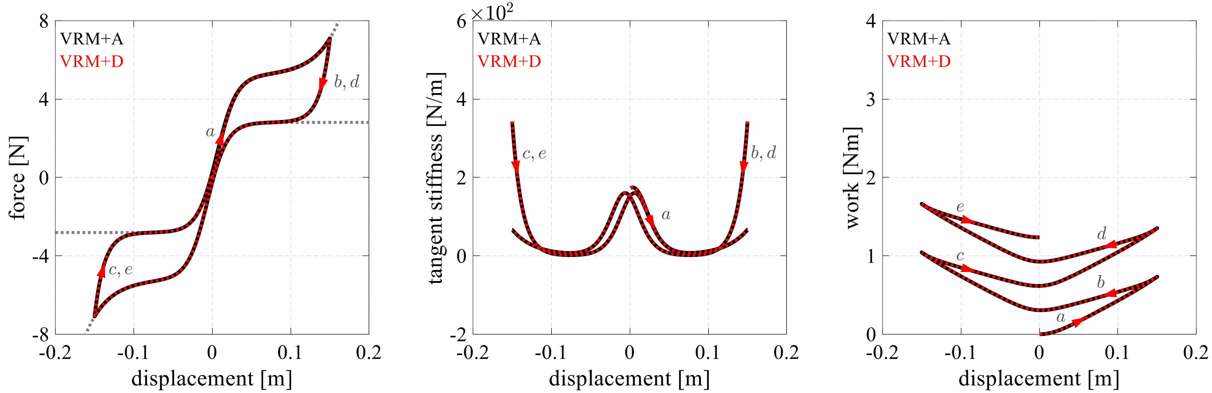
	k_b [$\mathrm{Nm^{-1}}$]	f_0 [N]	α [$\mathrm{m^{-1}}$]	β_1 [N]	β_2 [$\mathrm{m^{-1}}$]	γ_1 [N]	γ_2 [$\mathrm{m^{-1}}$]	γ_3 [m]
+	0	2.7	60	0	0	1	40	0.04
−	4	3	40	0	0	0.8	60	-0.10

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S4

$$m\ddot{u} + f = p(t)$$



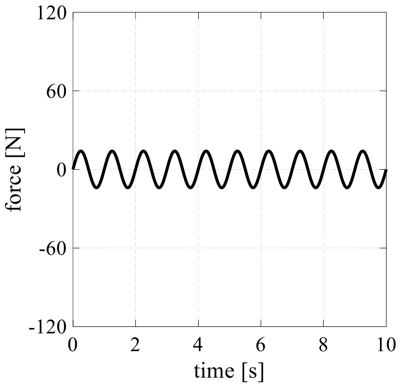
	k_b [Nm^{-1}]	f_0 [N]	α [m^{-1}]	β_1 [N]	β_2 [m^{-1}]	γ_1 [N]	γ_2 [m^{-1}]	γ_3 [m]
+	0	1.2	80	0.01	35	2	80	0.006
−	0	1.2	80	-0.01	-35	2	80	-0.006

Reformulation of the Vaiana-Rosati model

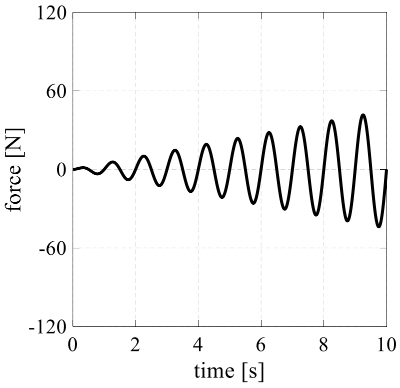
VRM+A versus VRM+D

Applied external forces

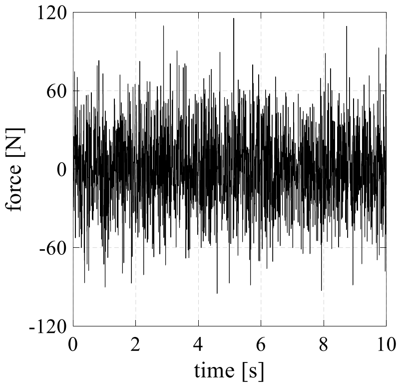
$$m\ddot{u} + f = p(t)$$



harmonic force with constant amplitude



harmonic force with increasing amplitude



random force

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S1

CEM-VRM+A

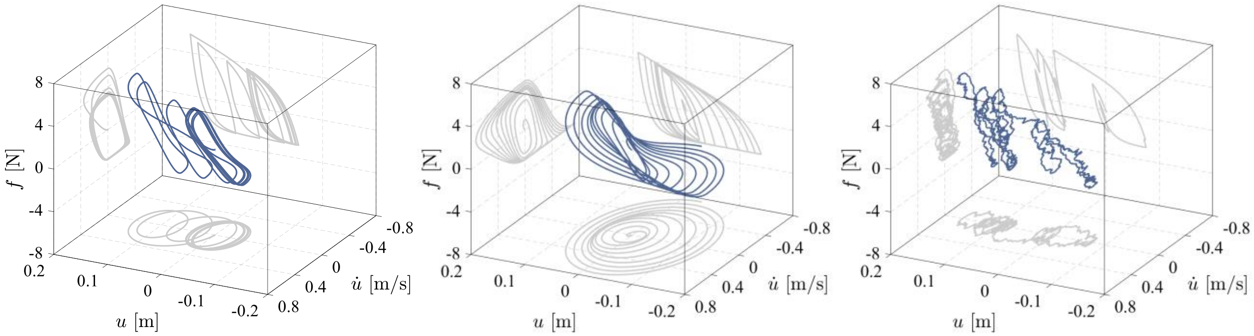
$$m\ddot{u} + f = p(t)$$

RKM-VRM+D

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = m^{-1}(p(t) - x_3)$$

$$\dot{x}_3 = \left[k_e(x_1) + k_b + \text{sign}(x_2)\alpha \times \left(f_e(x_1) + k_b x_1 + \text{sign}(x_2) f_0 - x_3\right)\right] x_2$$



harmonic force with constant amplitude

harmonic force with increasing amplitude

random force

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S2

CEM-VRM+A

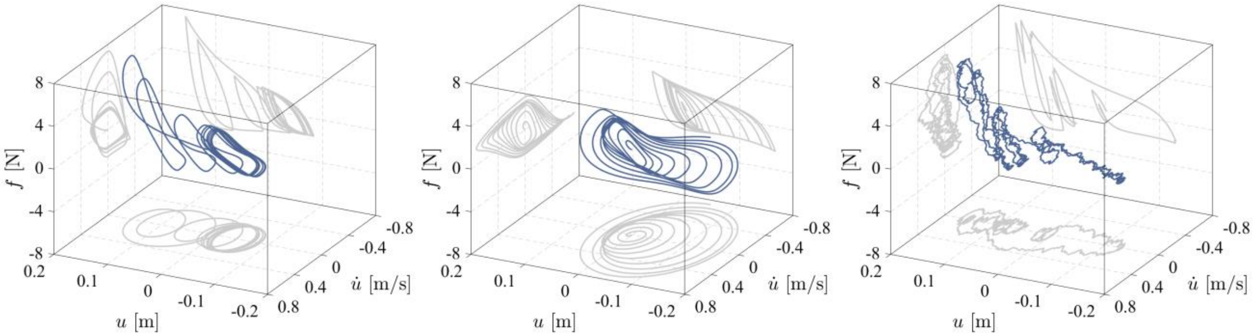
$$m\ddot{u} + f = p(t)$$

RKM-VRM+D

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = m^{-1}(p(t) - x_3)$$

$$\dot{x}_3 = \left[k_e(x_1) + k_b + \text{sign}(x_2)\alpha \times \left(f_e(x_1) + k_b x_1 + \text{sign}(x_2) f_0 - x_3\right)\right] x_2$$



harmonic force with constant amplitude

harmonic force with increasing amplitude

random force

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

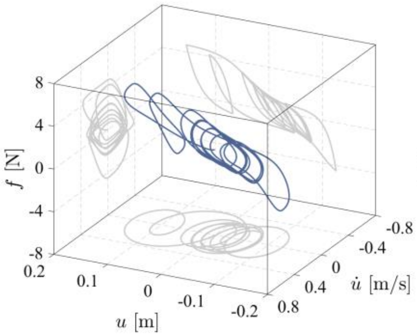
System S3

CEM-VRM+A

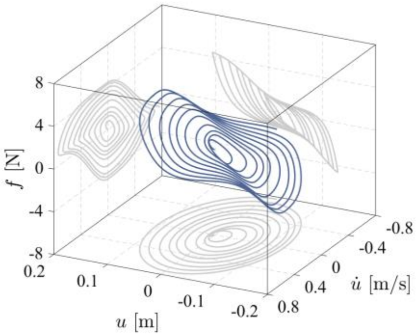
$$m\ddot{u} + f = p(t)$$

RKM-VRM+D

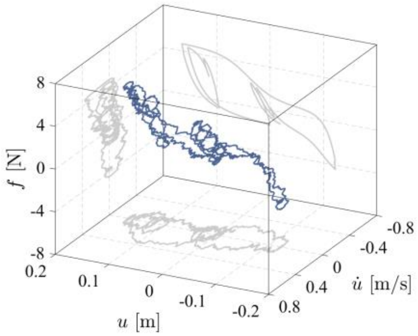
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = m^{-1}(p(t) - x_3)$$
$$\dot{x}_3 = \left[k_e(x_1) + k_b + \text{sign}(x_2)\alpha \times \left(f_e(x_1) + k_b x_1 + \text{sign}(x_2) f_0 - x_3\right)\right] x_2$$



harmonic force with constant amplitude



harmonic force with increasing amplitude



random force

Reformulation of the Vaiana-Rosati model

VRM+A versus VRM+D

System S4

CEM-VRM+A

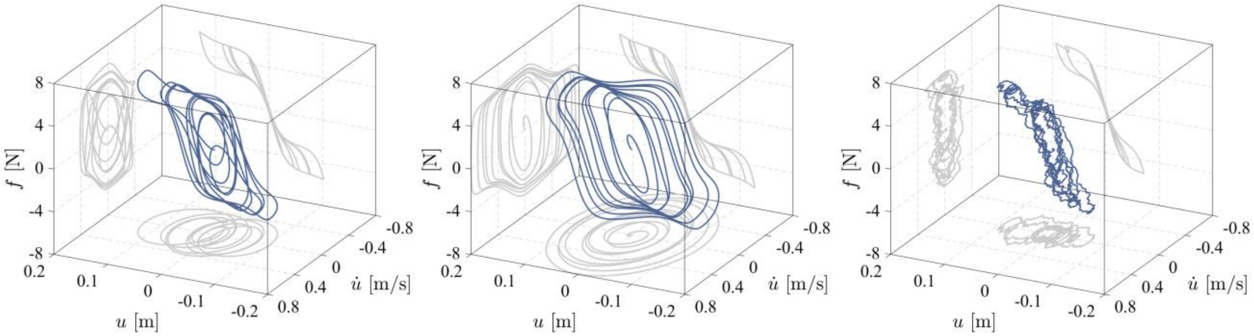
$$m\ddot{u} + f = p(t)$$

RKM-VRM+D

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = m^{-1}(p(t) - x_3)$$

$$\dot{x}_3 = \left[k_e(x_1) + k_b + \text{sign}(x_2)\alpha \times \left(f_e(x_1) + k_b x_1 + \text{sign}(x_2) f_0 - x_3\right)\right] x_2$$



harmonic force with constant amplitude

harmonic force with increasing amplitude

random force

References

[1] Vaiana N., Rosati L. (2023) Classification and unified phenomenological modeling of complex uniaxial rate-independent hysteretic responses. *Mechanical Systems and Signal Processing* 182: 109539.

[2] Vaiana N., Capuano R., Rosati L. (2023) Evaluation of path-dependent work and internal energy change for hysteretic mechanical systems. *Mechanical Systems and Signal Processing* 186: 109862.

[3] Vaiana N., Sessa S., Rosati L. (2021) A generalized class of uniaxial rate-independent models for simulating asymmetric mechanical hysteresis phenomena. *Mechanical Systems and Signal Processing* 146: 106984.

[4] Vaiana N., Sessa S., Marmo F., Rosati L. (2018) A class of uniaxial phenomenological models for simulating hysteretic phenomena in rate-independent mechanical systems and materials. *Nonlinear Dynamics* 93(3): 1647-1669.

Thank you for your Kind Attention

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